The Distance between Random Points in Two Rectangular Cities

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KEY WORDS: uniformly distributed random points; expected travel distance; pollution problem; transformation of variables

Abstract:

In modern city planning, cities are often designed in rectangular shape. The pollution (air pollutants, smog, noise, etc.) generated by the traffic going from one city to another is closely related to the distance between these two cities. To assess the distance between the two cities, two uniformly distributed random points, one from each city, are taken. Their travel distance (a straight path) is then measured. This travel plan is possible if both cities are interconnected with extensive road systems in which anywhere in one city can easily travel directly to anywhere in the other city. Other travel plans involving an exit point to a freeway in one city and an entry point from the same freeway in the other city will be considered in another paper.

Mathai (1998) considered the pollution problem created by vehicles travelling from suburbs to city core. He dealt with the situation when suburbs and city core are of circular shape. For uniformly distributed random points, he worked out various travel plans and the associated expected travel distances, along with the general moments and densities of the travel distances. He pointed out that the general expected travel distance for circular cities can be written in terms of an Appell's function or a Lauricella function. In this paper, the general expected travel distance for rectangular cities is presented explicitly and some numerical calculations are also given.

Motivated by some problems in survey sampling, Ghosh (1951) had provided explicit expressions for the probability density function of the distance between two random points in a rectangle, two adjacent rectangles and squares with a common diagonal. The results of the last two cases considered by Ghosh have been verified as special cases of the general result presented in this paper. Mathai et al. (1999) considered the distance between two independently and uniformly distributed random points in a cube. They obtained explicit forms of the exact density and arbitrary moments of the distance in terms of Gauss' hypergeometric functions and Appell's function.

In this paper, the probability density function of the distance between two uniformly distributed random points as well as its expected distance for rectangular cities are presented explicitly. The technique used here is the transformation of variables. In particular, the direct method of a four-variable transformation and the indirect method of two two-variable transformations will be used to obtain the desired explicit form for the density and its expected value. With the help of a computer, numerical calculations for computing the exact expected distance, the estimated distance and simulations are obtained for various situations. Finally, the integer moments of the travel distance will also be discussed.

References

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