

Rate of Convergence for the Nonparametric Maximum Likelihood Estimation of a Distribution Function Based on Doubly Interval-censored Failure Time Data

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Abstract:

The doubly interval-censored data arise in many disease progression or epidemiological studies. It is often the case that the survival time of interest is defined as the elapsed time between two related events, called initial and subsequent events, such as infection causing a disease and the onset of the disease. Many authors have studied the nonparametric estimation of a failure time distribution for the right-censored and interval-censored data in which the occurrence time of the initial event can be exactly observed and observations on the subsequent event are right-censored or interval-censored the survival time of interest (see DeGruttola and Lagakos, 1989; Gómez and Lagakos, 1994; Sun, 1995). The recent interests, however, has been to perform the statistical inference to the doubly interval-censored data. Fang and Sun (2001) obtained the consistency of the nonparametric maximum likelihood estimation (NPMLE) of a distribution function based on doubly interval-censored failure time data. In the meantime, they also proposed that the convergence rate of NPMLE for doubly interval censored data should not be faster than $n^{1/3}$, which is the convergence rate of NPMLE for the interval censored data (see Groeneboom, 1996).

The aim of the present paper is to solve the issue on the convergence rate of NPMLE for doubly interval-censored data. The nonparametric maximum likelihood estimation of a failure time distribution function based on doubly interval-censored data is studied and its rate of convergence is obtained, which is slower than that for the case of singly interval-censored data and thus give the positive answer to the question proposed by Fang and Sun (2001).

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