Moments for the Lowest Crossing, Pioneering, and Pivotal Sites in Critical Percolation

G. J. MORROW, *University of Colorado, USA*, E-mail: gjmorrow@math.uccs.edu
Y. Zhang, *University of Colorado, USA*

Key words: percolation, triangular lattice, lowest crossing, pivotal sites

Mathematical Subject Classification: Primary: 60K35, Secondary: 82B43

Abstract:

Let L_n denote the lowest crossing of a square $2n \times 2n$ box for critical site percolation on the triangular lattice imbedded in $\mathbf{Z}^2$. Denote also by F_n the pioneering sites extending below this crossing, and Q_n the pivotal sites on this crossing. The authors obtain the following moments of the sizes of these sets of vertices. For each positive integer τ, as $n \to \infty$,

1. $E(|L_n|^\tau) = n^{4\tau/3+o(1)}$
2. $E(|F_n|^\tau) = n^{7\tau/4+o(1)}$
3. $E(|Q_n|^\tau) = n^{3\tau/4+o(1)}$

These results extend to higher moments a discrete analogue of the recent results of Lawler, Schramm, and Werner [3] that the frontier, pioneering points, and cut points of planar Brownian motion have Hausdorff dimensions respectively 4/3, 7/4, and 3/4. The proof uses the recent results of Smirnov and Werner [4] on asymptotic probabilities of multiple arm paths in both the plane and half-plane and Kesten's connection method [2]. Kesten's method [1] handles the case of one-arm paths, and by Smirnov and Werner [4] may be extended to two and three-arm paths. The authors construct a new method for the estimation of the moments of pivotal sites.

References

[1] Kesten, H. (1986). The Incipient Infinite Cluster in Two-dimensional Percolation, *Probab Th Rel. Fields*, 73, 369-394.

[2] Kesten, H. (1987).Scaling Relations for 2D-Percolation, *Comm. Math. Phys.*, 109, 109-156.

[3] Lawler, G.F., Schramm, O. and Werner, W. (2001). The Dimension of the Planar Brownian Frontier is 4/3, *Math. Res. Lett.*, 8, 401-411.

[4] Smirnov, S. and Werner, W. (2001). Critical Exponents for Two-dimensional Percolation, *Math. Res. Lett.*, 8, 729-744.