Two-parameter p, q-variation Path and Integration of Local Times

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Abstract: In this paper, we prove two main results. The first one is to give a new condition for the existence of two-parameter p, q-variation path integrals and dominated convergence results for both the one-parameter and two-parameter integrals. Our condition of locally bounded p, q-variationis more natural and easy to verify than those of Young. The second result is to define the integral of local time $\int_{-\infty}^{\infty}\int_0^t g(s,x)d_{s,x}L_s(x)$ pathwise and then give generalized Itô's formula when $\nabla^- f(s,x)$ is only of bounded p, q-variation in (s,x). In the case that $g(s,x) = \nabla^- f(s,x)$ is of locally bounded variation in (s,x), the integral $\int_{-\infty}^{\infty}\int_0^t \nabla^- f(s,x)d_{s,x}L_s(x)$ is the Lebesgue-Stieltjes integral and was used in Elworthy, Truman and Zhao (2004). When $g(s,x) = \nabla^- f(s,x)$ is of only locally p, q-variation, where $p \geq 1$,$q \geq 1$, and $2q + 1 > 2pq$, the integral is a two-parameter rough path integral rather than a Lebesgue-Stieltjes integral. In the special case that $f(s,x) = f(x)$ is independent of s, we give a new condition for Meyer's formula and $\int_{-\infty}^{\infty} L_t(x)d_x\nabla^- f(x)$ is defined pathwise as a locally bounded one-parameter p-variation path integral. Both results are new in rough path theory and local time integration respectively. This is a joint work with Chunrong Feng.