Likelihood Ratio Tests: Old and New

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Abstract: The theory on the distribution of maximum likelihood ratios is fundamental and indispensible to classical parametric inferences. Despite their success in parametric inferences, the maximum likelihood ratio statistics might not exist in nonparametric function estimation setting. Even if they exist, they may be are hard to find and can be not optimal. The generalized likelihood statistics will be introduced to overcome these drawbacks. New Wilks' phenomenon is unveiled in infinite dimensional parameter spaces. We demonstrate that the generalized likelihood statistics (GLR) are asymptotically distribution free and follow χ^2-distributions for a number of testing problems and a variety of useful semiparametric and nonparametric models. These include the Gaussian white noise model, nonparametric regression models, varying coefficient models, additive models and partial linear varying-coefficient models. We further demonstrate that generalized likelihood ratio statistics are asymptotically optimal in the sense that they achieve optimal rates of convergence given by Ingster (1993). They can even be adaptively optimal in the sense of Spokoiny (1996). Issues on bias reduction will be addressed. The talk is based on a series of recent papers by my collaborators, Chunming Zhang, Jian Zhang, Jiangcheng Jiang and Tao Huang.