

Harnack Inequalities, Rademacher's Theorem and Transportation Inequalities on Loop Groups

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Abstract: We consider a Riemannian distance on loop groups. A dimension-free Harnack inequality is established on loop groups with respect to this distance. Then we prove a Rademacher's type theorem for Lipschitz continuous functions relative to this Riemannian distance. In the end, by the method of Hamilton-Jacobi semigroup used in [9], transportation cost inequalities are established on loop groups.

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