

# Estimating Stochastic Volatility with Noisy High Frequency Data

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**Abstract:** With the availability of high frequency financial data, nonparametric estimation of volatility of an asset return process becomes feasible. A major problem is how to estimate the volatility consistently and efficiently, when the observed asset returns contain error or noise, for example, in the form of microstructure noise (Zhou (1996), Corsi, Zumbach, Muller, and Dacorogna (2001), Ait-Sahalia, Mykland and Zhang (2003), Zhang, Mykland and Ait-Sahalia (2003), Hansen and Lunde (2004), Zhang (2004)). In this talk, we shall show, both analytically and empirically, why and how the customary way of estimating volatility is awed. We propose a new estimator based on subsampling and averaging over different time scales. Our estimator is consistent, efficient, and asymptotically normal. It converges to the true volatility at the rate of  $n^{-1/4}$ . which is the best attainable. The estimator remains valid when the observation noise is dependent.

## References

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