

# Remarks on Central Limit Theorems for the Number of Percolation Clusters

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KEY WORDS: percolation, central limit theorem, tree, Sierpiński carpet

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 82B43; secondary 60F05, 28A80

## Abstract:

The central limit theorems (CLT's) for some quantities related to Bernoulli percolation problems are studied by many authors. For Bernoulli bond percolation problem on  $\mathbf{Z}^d$  ( $d \geq 2$ ), Zhang[3] proved a CLT for the number of open clusters in a finite box for all  $p \in (0, 1)$ , that is, including the case  $p = p_c(\mathbf{Z}^d)$ . Using his method together with the ergodic theorem, Penrose[2] proved a general CLT which can be applied to several models.

We study limit theorems for percolation problems on pre-fractal graphs and trees, where the ergodic theorem is not available. For a class of Sierpiński carpet lattices, considered by Kumagai[1], we obtain some limit theorems for the number of percolation clusters. We can also prove a CLT for the number of percolation clusters on trees of bounded degree.

## References

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