Shape Restricted Estimation in the Search for Dark Matter

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Abstract: In addition to visible matter, like stars, there is matter that astronomers cannot see. They know that it is there from gravitational effects. It's nature and even location are not well known at present, but there is a wide belief that the vast majority of matter in the Universe is dark. The talk centers on statistical issues that arise in the non-parametric estimation of dark matter in the dwarf spheroidal galaxies that are satellites of the Milky Way. Statistically, this is an inverse problem with missing data. The distribution of (total) mass together with the law of gravity determines the positions and velocities of stars, and the statistical problem is to recover the distribution of mass from observed positions and velocities. Complete positions and velocities cannot be observed here–only the radial velocity and the projection of position on the plane orthogonal to the line of sight. The distribution of projected position can be estimated quite precisely from large ($\sim 150,000$ stars) photometric studies. Velocities require some time to measure, however, and previous studies have been too small (~ 30 stars) to allow estimation in any generality. Instead, astronomers have been forced to adopt highly structured parametric models. New instrumentation will soon make much larger samples (~ 1500 stars) available and non-parametric methods feasible. Assuming spherical symmetry, let $M(r)$ denote the total mass within r of the center of a dwarf spheroidal galaxy. Then Jeans' Equation (from classical potential theory) shows that $M(r) = c\Psi''(r)/f(r)$, where c is a known constant, f is a (nearly) known non-negative function, and Ψ is a function for which an unbiased, $\sqrt{n}$-consistent estimator exists. From Jeans' Equation, Ψ is a convex function, and $\Psi''(r)/f(r)$ is non-decreasing in r. The unbiased estimator, $\Psi^{\#}$ say, does not satisfy these shape restrictions and can be improved by imposing them. The talk centers on methods for accomplishing this, and properties of the resulting estimators. This is joint work with Xiao Wang (UM Statistics) and Mario Mateo and Matt Walker (UM Astronomy).