

A Haar-fisz Technique for Locally Stationary Volatility Estimation

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Abstract:

We propose a locally stationary model for financial log-returns whereby the returns are independent and the volatility is a piecewise constant function with an unknown number and location of jumps, defined on a compact interval to enable a meaningful estimation theory. We demonstrate that our model explains well the common stylised facts of log-returns. We propose a new wavelet thresholding algorithm for volatility estimation in our model, where Haar wavelets are powerfully combined with the variance-stabilizing Fisz transform. The resulting volatility estimator is mean-square consistent with a near-parametric rate, does not require any pre-estimates, is rapidly computable and easy to implement. We also discuss important variations on the choice of estimation parameters.

In an extensive empirical study, we fit our model to a variety of currency exchange datasets, and identify versions of our algorithm which perform particularly well in producing “well-behaved” empirical residuals. We show that our algorithm significantly outperforms GARCH in long-term volatility forecasting, and performs slightly better than a model-free moving window technique. We also demonstrate that the short-term volatility forecasting ability of our algorithm matches that of the GARCH(1,1) technique. In summary, by using our methodology, we are able to obtain both good fits to the data and convincing short- and long-term volatility forecasts.

Preliminary investigation suggests that our Haar-Fisz methodology can also be used if the returns are modelled as correlated.

References

- [1] Fryzlewicz P., Sapatinas T. and Subba Rao S. (2004). A Haar-fisz Technique for Locally Stationary Volatility Estimation, *Submitted For Publication*, URL: http://www.ma.imperial.ac.uk/~pzf/hf_vol/hf_vol.pdf or http://www.ma.imperial.ac.uk/~pzf/hf_vol/hf_vol.ps.