# Analyzing Proportions: Given Prior as A Beta Parametric Model and Moments

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KEY WORDS: Proportion, Optimization, Chi-square divergence, Beta distribution, Moments

MATHEMATICAL SUBJECT CLASSIFICATION: 60E05, 62H05

**Abstract**:

The minimum $\chi^2-$divergence principle [1] states: *When a prior probability density function of $X$ , $g(x)$, which estimates the underlying probability density function $f(x)$ is given in addition to some constraints, then among all the density functions $f(x)$ which satisfy the given constraints we should select that probability density function which minimizes the $\chi^2-$divergence* . We consider the applications of this principle in analyzing the proportions given a prior parametric model for proportions as the beta distribution and partial information on moments, in particular on mean and variance. Thus, having selected (diagnosed) the model for a given application, we provide an appropriate method to characterize and analyze the data set. The main result is:

*Theorem*: Let given be a prior *beta* probability distribution of $X$ with the density function.

$$g(x) = \frac{x^{p-1}(1-x)^{q-1}}{B(p,q)} \ , p, q > 0, 0 < x < 1, \tag{1}$$

where

$$B(p,q) = \int_0^1 u^{p-1}(1-u)^{q-1}du, \tag{2}$$

and the constraints

$$f(x) \geq 0, \int f(x)dx = 1, \ \int [h(x)]^t f(x)dx = m_{t,f}, \ t = 1, 2, 3, ..., r. \tag{3}$$

Then, the minimum $\chi^2-$divergence probability distribution of $X$ has the density function

$$f(x) = \frac{x^{p-1}(1-x)^{q-1}}{2B(p,q)} \left( \alpha_0 + \sum_{t=1}^{r} [h(x)]^t \alpha_t \right), \tag{4}$$

and the $(r+1)$ constants, $\alpha_0$ and $\alpha_t$, $t = 1, 2, 3, ..., r$, are determined from

$$\int \frac{x^{p-1}(1-x)^{q-1}}{2B(p,q)} \left( \sum_{t=1}^{r} [h(x)]^t \alpha_k \right) dx = 1 - \frac{\alpha_0}{2}, \tag{5}$$

and

$$\int \frac{x^{p-1}(1-x)^{q-1}}{2B(p,q)} [h(x)]^t \left( \alpha_0 + \sum_{t=1}^{rt} [h(x)]^t \alpha_t \right) dx = m_{t,f}. \tag{6}$$

The minimum $\chi^2-$divergence:

$$\chi^2_{\min}(f,g) = \int \frac{x^{p-1}(1-x)^{q-1}}{4B(p,q)} \ \left( \alpha_0 + \sum_{t=1}^{r} [h(x)]^t \alpha_t \right)^2 dx - 1. \tag{7}$$

## References

[1] Kumar, P. and Taneja, I.J. (2005). Chi-square Divergence and Minimization Problem for Continuous Probability Distributions, *Preprint*.