Limiting Distributions of Non-central t-Statistics

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Abstract: Let $X, X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables. Define $\mathbb{T}_n = \sqrt{n}\bar{X}/S$, where $\bar{X}$ and S^2 are the sample mean and the sample variance, respectively. We refer to $\mathbb{T}_n$ as central or non-central (Student's) t-statistics, or simply t-statistics, depending on $\mathbf{E}X = 0$ or $\mathbf{E}X \neq 0$, respectively. Non-central t-statistics arise naturally in the calculation of powers for t-tests. Central t-statistics have been well studied while there is a very limited literature on non-central t-statistics. In this paper, we attempt to narrow this gap by studying the limiting behavior of non-central t-statistics, which turns out to be much more complicated. For instance, it is well known that, under finite second moment conditions, the limiting distributions for central t-statistics are normal while those for non-central t-statistics can be non-normal, and critically depend on whether or not $\mathbf{E}X^4 = \infty$. As an application, we study the effect of non-normality on the performance of the Student's t-tests. This talk is based on a joint work with V. Bentkus, B.Y. Jing and W. Zhou.