Strong Stability of Weighted Sums of NA Random Variables

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Abstract:

In this paper we study the almost sure(strong) stability of weighted sums of NA random variables. We prove the following main results.

Theorem 1 Let $\{a_n, n \geq 1\}$ and $\{b_n, n \geq 1\}$ be two sequences of positive numbers with $c_n = b_n/a_n$ and $b_n \uparrow \infty$. Let $\{X_n, n \geq 1\}$ be a sequence of NA random variables which is stochastically dominated by a non-negative random variable X. Set $N(x) =$Card$\{n : c_n \leq x\}, x > 0$. $1 \leq p \leq 2$. If the following conditions are satisfied:
(1) $EN(X) < \infty$,
(2) $\int_0^\infty t^{p-1} P(X > t) \int_t^\infty N(y)/y^{p+1} dydt < \infty$,
then there exist $d_n \in R, n = 1, 2, \cdots$ such that

$$b_n^{-1} \sum_{i=1}^{n} a_i X_i - d_n \to 0 \quad a.s..$$

Theorem 2 Let $\{X_n, n \geq 1\}$ be a sequence of identically distributed NA random variables. If $E|X_1|\alpha(\log^+ |X_1|) < \infty$, then there exist $d_n \in R, n = 1, 2, \cdots$ such that $b_n^{-1} \sum_{i=1}^{n} a_i X_i - d_n \to 0 \quad a.s..$

Theorem 3 Let $\{X_n, n \geq 1\}$ be a sequence of NA random variables(not necessarily identically distributed). $1 \leq P \leq 2$. If $\sum_{n=1}^{\infty} n^{-p} E|X_n \alpha(\log^+ |X_n|)|^p$, then there exist $\{d_n\} \subset R$ such that $b_n^{-1} \sum_{i=1}^{n} a_i X_i - d_n \to 0 \quad a.s..$