Estimate The Association In Clayton Model

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Abstract:

Bivariate survival time data arise when a sample consists of two variables. The analysis of bivariate survival time must reflect the non-independence of failures within the two variables. Let T_1 and T_2 be survival times with continuous joint probability density $f(t_1, t_2)$. Clayton model defined by the following formula, for all s, t such that $f(s,t) > 0$,

$$f(s,t)\int_s^\infty \int_t^\infty f(u,v)dvdu = \theta \int_s^\infty f(u,t)du \int_t^\infty f(s,v)dv. \tag{1}$$

is a appealing representation for such data. The parameter θ, called association parameter, measures the degree of association between T_1 and T_2. Independence of T_1 and T_2 is implied by $\theta = 1$ and positive association is implied by $\theta > 1$. Inverse association is implied by $\theta < 1$, but this case seems to have little practical importance (see [1]).

Several papers have proposed semi-parametric inference procedures about θ without specification of F_1 and F_2. [2] proposed a semi-parametric estimator of θ based on concordance estimator of [3] and evaluated its asymptotic variance. [1] and [4] investigated a weighted concordance estimator which gives higher weight to late failures. But the large-sample properties is little known. [5] and [6]estimated θ from a pseudo likelihood with nonparametric estimation of the marginal survival functions. [7] extended the above approach and proposed a two-stage estimator of θ. This estimator is consistent and asymptotically normal under mild regularity conditions.

Importance and useful applications of Clayton model under right censorship can be found in [7,8]. In this talk we give an estimator of θ under right censorship. The estimator is proved to be consistent and its asymptotic distribution is shown to be normal. The asymptotic variance is given explicitly. The estimator is easy to calculate. Simulations assess the performance of the estimator.

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