Two-dimensional Poisson Trees Converge to the Brownian Web

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Abstract: The *Brownian web* can be roughly described as a family of coalescing one-dimensional Brownian motions starting at all times in R and at all points of R. It was introduced by Arratia, [1,2]; a variant was then studied by Tóth and Werner,[6]; another variant was analyzed recently by Fontes, Isopi, Newman and Ravishankar, [4,5]. The two-dimensional *Poisson tree* is a family of continuous time one-dimensional random walks with uniform jumps in a bounded interval. The walks start at the space-time points of a homogeneous Poisson process in R^2 and are in fact constructed as a function of the point process. This tree was introduced by Ferrari, Landim and Thorisson, see [3]. By verifying criteria derived by Fontes, Isopi, Newman and Ravishankar [4,5], we show that, when properly rescaled, and under the topology introduced by those authors, Poisson trees converge weakly to the Brownian web.

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