Discontinuous Linear Statistics in the Gaussian Unitary Ensembles

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Abstract:

Linear statistics are an important statistical tool usually expressed as

$$\Phi := \sum_{j=1}^{n} \phi(x_j).$$

where $x_1, .., x_n$ are n random variables with p.d.f. $p(x_1, .., x_n)$ Its distribution functions are of particular interest. For the Gaussian Unitary Ensembles $p(x_1, .., x_n)$ is a constant multiple of

$$\prod_{1 \le j < k \le n} (x_j - x_k)^2 \exp(-\sum_{l=1}^{n} x_l^2),$$

where $-\infty < x_j < \infty$.

It is well known that the computation of the distribution function of Φ reduces to the determination of polynomials orthogonal w.r.t. $\exp(-x^2)$, namely, the Hermite polynomials. Furthermore if $\phi(x)$ is a "nice" function then a lot is known about the distribution of Φ, when n is large. However, if $\phi(x)$ has discontinuities then the problem becomes difficult.

I will show in this talk that by incorporating the jump into the original weight, the new weight

$$w(x) = \exp(-x^2) w_J(x, t)$$

produces a new class of orthogonal polynomials where the diagonal recurrence coefficients considered as function of t satisfies is a particular Painleve IV transcendent for fixed n.