Geometric Process and Its Application

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Abstract:

Lam [1,2] introduced the following geometric process.

Definition. A stochastic process $\{X_n, n = 1, 2, \ldots\}$ is a geometric process (GP), if there exists some $a > 0$ such that $\{a^{n-1}X_n, n = 1, 2, \ldots\}$ forms a renewal process. The number a is called the ratio of the geometric process.

Clearly, a GP is stochastically increasing if the ratio $0 < a \leq 1$; it is stochastically decreasing if the ratio $a \geq 1$. A GP will become a renewal process if the ratio $a = 1$. Thus, the GP is a simple monotone process and is a generalization of renewal process.

Let $E(X_1) = \lambda$ and $Var(X_1) = \sigma^2$, then

$$\begin{aligned} E(X_n) &= \frac{\lambda}{a^{n-1}}, \\ Var(X_n) &= \frac{\sigma^2}{a^{2(n-1)}}. \end{aligned}$$

Therefore, a, λ and σ^2 are three important parameters in the GP.

In this talk, we shall introduce the fundamental probability theory of a GP, then consider the hypothesis testing of the GP and estimation problem of three important parameters in the GP. On the other hand, the application of GP to reliability, especially to the maintenance problem is studied. Furthermore, the application of GP to the analysis of data from a sequence of events with trend is investigated. From real data analysis, it has been shown that on average, the GP model is the best one among four models including a Poisson process model and two nonhomogeneous Poisson process models. See Lam (2005) for a brief review and more references.

References

[1] Lam, Y. (1988). A Note on the Optimal Replacement Problem, *Adv. Appl. Prob.*, 20, 479-782.

[2] Lam, Y. (1988). Geometric Processes and Replacement Problem, *Acta Math. Appl. Sinica*, 4, 366-377.

[3] Lam, Y. (2005). Geometric Process, to appear in *The Encyclopedia of Statistical Sciences, 2nd edition, N. Balakrishnan, C. Read, S. Kotz and B. Vidakovic, ed. John Wiley & Sons, Inc., New York.*