Effect Reversal, Collapsibility and Decomposability for Causal Inference

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Abstract:

Association between two variables may be reversed by marginalizing over another possibly unobserved background. This reversal is called the Yule-Simpson paradox (Yule, 1903; Simpson, 1951). To avoid the reversal, many authors discussed collapsibility of association measures over a background (Wermuth, 1987; Geng, 1992; Cox and Wermuth, 2003). Causal effects and relationships among variables may be confounded by ignoring other variables (Pearl, 2000; Greenland, Robins and Pearl, 1999; Geng, Guo and Fung 2002). Directed acyclic graphs (DAGs) are widely used to represent independence, conditional independence and causal relationships among the variables (Spirtes, Glymour and Scheines, 1993; Cox and Wermuth, 1996; Lauritzen, 1996; Pearl, 2000). Structure recovery of DAGs has been discussed by many authors (Spirtes, Glymour and Scheines, 1993; Cowell et al., 1999; Pearl, 2000).

In this paper, we show the condition for collapsibility of the association measure and we discuss local structure recovery for a DAG after marginalization over unobserved variables. This problem of localization is related to problems of identification and collapsibility. We present a condition for this localized recovery and explain which edges and directions of edges in local structure can be recovered validly from the marginal distribution, which edges may be spurious and which directions may become to be indeterminable after marginalization. Further, we propose that structural learning of a directed acyclic graph can be decomposed into problems related to its decomposed subgraphs. The decomposition of structural learning requires conditional independencies, but it does not require that separators are complete subgraphs. Domain or prior knowledge of conditional independencies and incompletely observed data patterns can be utilized to facilitate the decomposition of structural learning. This decomposition can also be used to discover local causal relationships.

References

[1] Cowell, R.G., Dawid, A.P., Lauritzen, S.L. and Spiegelhalter, D.J. (1999). Probabilistic Networks and Expert Systems, Springer Publications, New York.

[2] Cox, D.R. and Wermuth, N. (2003). A General Condition for Avoiding Effect Reversal after Marginalization, *J. R. Statist. Soc., B*, 65, 937-941.

[3] Cox, D.R. and Wermuth, N. (1996). Multivariate Dependencies: Models, Analysis, and Interpretation, Chapman and Hall, London.

[4] Geng, Z. (1992). Collapsibility of Relative Risks in Contingency Tables with a Response Variable, *J. R. Statist. Soc., B*, 54, 585-593.

[5] Geng, Z., Guo, J. and Fung, W.K. (2002). Criteria for Confounders in Epidemiological Studies. *J. Royal Statist. Soc., B*, 64, 3-15.

[6] Greenland, S., Robins, J.M. and Pearl, J. (1999). Confounding and collapsibility in causal inference, *Statist. Sci.*, 14, 29-46.

[7] Lauritzen, S.L. (1996). Graphical models, Clarendon Press, Oxford.

[8] Pearl, J. (2000). Causality, Cambridge University Press, Cambridge.

[9] Simpson, E.H. (1951). The Interpretation of Interaction in Contingency Tables. *J. R. Statist. Soc., B*, 13, 238-241.

[10] Spirtes, P., Glymour, C. and Scheines, R. (1993). Causation, Prediction and Search, Springer-Verlag, New York.

[11] Wermuth, N. (1987). Parametric Collapsibility and the Lack of Moderating Effects in Contingency Tables with a Dichotomous Response Variable, *J. R. Statist. Soc., B*, 49, 353-364.

[12] Yule, G.U. (1903). Notes on the Theory of Association of Attributes in Statistics, *Biometrika*, 2, 121-134.