Memoire of the International Symposium on Approximation Theory and Remote Sensing Applications

The ISATRSA was organized by

- Institute of Remote Sensing Applications, Chinese Academy of Sciences (CAS);
- School of Mathematical Sciences, Beijing Normal University (BNU);
- Department of Mathematics, Yunnan Nationalities University (YNaU)

The organizing committee:

- Chairman Professor Xu Lizhi (Dalian)
- Co-Chairman Professor Wang Kunyang (BNU)
- Co-Chairman Professor Liu Yongping (BNU)
- Co-Chairman Professor Tang Ping (CAS)
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- Secretaries:
  Mr. Huang Hongwei (BNU),
  Ms. Xiao Weiwei (BNU),
  Mr. Wang Yi (CAS),
  Ms. Xiong Shunqing (YNaU)

The symposium was held from April 22 to 24, 2006 in the City Kunming, Yunnan Province of China. There were totaly 57 participants who were from China, Australia, Canada, Russia, Israel, Austria.

Eleven plenary lectures and some poster research papers were presented. The topics discussed concerned classical approximation, spherical harmonic analysis, wavelets, learning theory, image processing, remote sensing inverse problem, etc.

The plenary speakers are:

Professor Gavin Brown (University of Sydney, Australia),

Professor Zeev Ditzian (University of Alberta, Canada),

Professor B. Kashin (Academy of Science of Russia),

Professor Peng Lizhong (Peking University, China),

Professor Sergei V.Pereverzyev (Johann-Radon-Institute (RICAM), Austria),

Professor Allan Pinkus (Technion - Israel Institute of Technology, Israel),

Professor Wang Kunyang (Beijing Normal University, China)

Dr. Wang Yanfei (Institute of Remote Sensing Applications, CAS, China),

Professor Xie Tingfan (China Jiliang University, China),

Professor Xu Shuzhan (Philips, Shenzhen, China),

Professor Zhou Songping (Zhejiang University of Technology, China),

The symposium was financially supported by

National Natural Science Foundation of China,

Beijing Normal University,

Institute of Remote Sensing Applications (CAS),

Yunnan Nationalities University.

Program of the Symposium

Place: Meeting Hall (2nd floor), Kunming Education Hotel

April 23, 2006, Morning

Opening ceremony, chaired by Professor Tang Ping

09:00—Opening speech by Professor Xu Lizhi

09:30—Speech by Mr. Yu Dalin,
Chief of Science and Technology Section, Yunnan Education Committee

09:45—Speech by Ms. Ma Lijuan,
the vice President of Yunnan Nationalities University

10:00—Coffee break, Taking group photo

Plenary session, chaired by Professor Xu Lizhi

10:50— Professor G. Brown:
Positivity and boundedness of trigonometric sums

April 23, 2006, Afternoon

Plenary session Chaired by Professor Wang Kunyang

14:00 Professor Z. Ditzian:
Measure of smoothness on the sphere

14:55 Professor Allan Pinkus:
Density in Approximation Theory

15:40 Coffee break

Plenary session, Chaired by Professor Z. Ditzian

16:00 Professor Sergei V.Pereverzyev:
Regularization algorithm in learning theory

16:55 Professor Wang Kunyang:
New results concerning the sphere obtained after 2001

17:40— Poster session

18:30 Banquet

April 24, 2006, Morning

Plenary session, Chaired by Professor Liu Yongping

08:30 Professor B.S.Kashin:

On some applications of approximation theory: image compression and error correcting codes

09:25 Dr. Wang Yanfei:

On the effective inversion method for retrieval of land surface parameters

Plenary session, Chaired by Professor B. Kashin

10:30 Professor Peng Lizhong:

Wavelets and image/video coding

11:25 Professor Xu Shuzhan:

An Approximate Algorithm to Solve Linear Systems by Matrix with Off-diagonal Exponential Decay Entries

April 24, 2006, Afternoon

Plenary session Chaired by Professor S. V.Pereverzyev

14:00 Professor Xie Tingfan:

A survey of Rational Approximation of Function $|x|$ and its Applications

14:55 Professor Zhou Songping:

Some New Conditions in Fourier Analysis and Applications

Plenary session, Chaired by Professor Allan Pinkus

16:00 Closing Speech by Professor Tang Ping

April 25, 2006 In-city visit and sight-seeingWorld Horti-Expo Garden etc.

List of the papers in the poster sessions

1. Cai Feng , Zhou Bo, Gu Ming, Effectiveness Study On Multi-Objective Nonlinear Optimization- Comparison of Niche Evolution Algorithm and Particle Swarm Optimization

2. Cao Hui, Balancing Principle for Elliptic Cauchy Problem

3. Chen Di-Rong , Sun Tao , Consistency of Multiclass Empirical Risk Minimization Methods Based on Convex loss

4. Guo Shunsheng, The Central Approximation Theorems for Baskakov-Bézier Operators

5. Min HAN and Di-Rong CHEN, Analysis to Neyman-Pearson classification with Convex Loss Function

6. Kuang Honghai, Machine learning in image processing of BESTMAP

7. Tatiana Kulikova, Some properties of approximation with frames

8. Liang Xue-Zhang, Cui Li-Hong, Zhang Jie-Lin, Zhang Ming, Multivariate Lagrange Interpolation and an Application of Cayley-Bacharach Theorem

9. Sheng Baohuai, On the Degree of Approximation by Spherical Translations

10. Wu Xiao-hong, Lu Zhi-kang, Approximation to Function $|x|^{\alpha}$ by Interpolating Polynomials

11. Xu Zhiqiang, Using Splines to Study Some Problems in Combinatorics: A Survey

12. Zeng Xiao-Ming, Convergence Properties of Control Meshes of Catmull-Clark Subdivision Surfaces

13. Zhang Chun-gou, Wang Quan-e, The Complete Asymptotic Expansion for the Baskakov Operators

Abstracts of the papers in poster sessions

Effectiveness Study On Multi-Objective Nonlinear Optimization- Comparison of Niche Evolution Algorithm and Particle Swarm Optimization

Feng Cai[1),2)], Zhou Bo[1)], Ming Gu[2)]

1) Dalian Naval Academy, Dalian, China 116018, vipcaif@tom.com

2) China ship scientific research center, Wuxi, China 214082

For complicated nonlinear optimum selection problems, the traditional Standard genetic algorithm (SGA) can get the global optimal solution when the parameters of SGA are well selected. For combined optimum problems with continuous variables, the code length of SGA is longer and the generation is higher to ensure a certain precision, so more time is needed for calculation. Further more, its very difficult to get all global solutions when there are more than one global optimum solution for the problem. In order to get over the shortcomings of Genetic Algorithm, a nonlinear optimal selection model based on niche evolution algorithm (NEA) is proposed in this paper, and the principal for parameters selection of NEA is also discussed. The NEA not only has the virtue of simplicity because the parameters of NEA cope directly with variables themselves, but also has the ability to get all global solutions effectively by adjusting the parameters properly. Meanwhile, Particle Swarm OptimizationPSO method also can track the optimal solution through the collaboration of the particles in the swarm, because the particles share the population intelligence, it is also a parallel global searching method which comprises a very simple concept and is computationally inexpensive in terms of both memory requirements and speed. So we compare NEA with PSO to study the effectiveness of these two methods on multi-objective nonlinear optimization, including the time consumpation, optimization precision, robustness and the ability of gloable multi-objective optimization. A plentiful of simulations have been done, and the results shows that both of these two methods have good ability for multi-objective nonlinear optimization, and NEA method is more simple and effective.

Keywords: Niche Evolutionary Algorithm, Nonlinear Optimization, Partial Swarm Optimization, Multi-objective

Balancing Principle for Elliptic Cauchy Problem

Cao Hui

A classical ill-posed problem, elliptic Cauchy Problem is considered. Discretization is applied as a regularization method to obtain a stable approximate solution. Using balancing principle, we propose an adaptive strategy for the choice of appropriate discretization parameter (also can be considered as regularization parameter). Numerical tests illustrate the theoretical results.

Consistency of Multiclass Empirical Risk Minimization Methods Based on Convex loss[1]

Di-Rong Chen [1)], Tao Sun[2)]

1) Department of Applied Mathematics,
Beijing University of Aeronautics and Astronautics,
Beijing 100083, P. R. CHINA
Email: drchen@buaa.edu.cn

2) Email: st.126@163.com.cn

The consistency of classification algorithm plays an central role in statistical learning theory. A consistent algorithm guarantees us that taking more samples essentially suffices to roughly reconstruct the unknown distribution. We consider the consistency of empirical risk minimization (ERM) algorithm based on convex loss in multiclass classification. Our approach is, under some mild conditions, to establish a quantitative relationship between classification errors and convex risks. Unlike previous work, the conditions are expressed in terms of the convex loss function. The above relationship is used to establish the consistency of ERM scheme over classes of combinations of very simple rules (base classifiers). Finally, an AdaBoost algorithm for multiclass classification is proposed as a down-the-gradient method for implementing ERM with exponential loss over class of combinations of base classifiers. It is a straightforward generalization of AdaBoost to multiclass using multiclass classifiers. One of its advantages is that it works even when the base classifier can not achieve 50% accuracy, in which AdaBoost.M1 fails.

[1]Research supported in part by NSF of China under grants 10571010 and 10171007

The Central Approximation Theorems for Baskakov-Bézier Operators

Shunsheng Guo

College of Mathematics and Information Science, Hebei Normal University
Shijiazhuang 050016, People's Republic of China
E-mail: ssguo@hebtu.edu.cn

For Baskakov-Bézier operators we give the direct, inverse and equivalence approximation theorems with unified Ditzian-Totik modulus $\omega_{\varphi^\lambda}(f,t)$ $(0 \leq \lambda \leq 1)$.

Keywords: Baskakov-Bézier operator; Ditzian-Totik modulus; K-functional; Approximation equivalent theorem

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Analysis to Neyman-Pearson classification with Convex Loss Function [2]

Min HAN Di-Rong CHEN

Department of Mathematics, and LMIB
Beijing University of Aeronautics and Astronautics 100083
Email: haaam@163.com drchen@buaa.edu.cn

Neyman-Pearson classification has been studied in several articles before. But they all proceeded in the classes of indicator functions with indicator function as the loss function, which make the calculation to be dicult. This paper investigates Neyman-Pearson classication with convex loss function in the arbitrary class of real measurable functions. A general condition is given under which Neyman-Pearson classication with convex loss function has the same classier as that with indicator loss function. We give analysis to NP-ERM with convex loss function and prove its performance guarantees. An example of complexity penalty pair about convex loss function risk in terms of Rademacher averages is studied, which produces a tight PAC bound of the NP-ERM with convex loss function.

Keywords: Neyman-Pearson lemma, convex loss function, Neyman-Pearson classification, NP-ERM, Rademacher average.

MR(2000) Subject Classification 62G05; 68T05; 68T10

[2]Research supported in part by NSF of China under grants 10571010 and 10171007

Machine learning in image processing of BESTMAP

Honghai Kuang

Southwest University, Beibei, Chongqing, China, 400715

The paper discussed the image processing of bestmap and wanted to set up knowledge representation of bestmap and improve the speed of bestmap.

Keywords: machine learning; image processing; bestmap

Some properties of approximation with frames

Tatiana Kulikova

Russian Academy of Sciences Publishing Center, Moscow, Russia
Kulikova@mi.ras.ru

Definition Let H be a separable Hilbert space, $\phi = \{\varphi_j\}_{j=1}^{\infty} \subset H$, $\varphi_j \neq 0$, $\forall j$. System ϕ is a frame, if

$$A\|v\|^2 \leq \sum_{j=1}^{\infty}(v, \varphi_j)^2 \leq B\|v\|^2,$$

$\forall v \in H$. (Here A and B are absolute constants.)

ϕ is a frame and ϕ is a minimal set $\Longleftrightarrow$ ϕ is a Riesz basis.

Theorem (Kashin B., Kulikova T.) $\phi = \{\varphi_j\}_{j=1}^{\infty} \subset H$ is a frame $\Longleftrightarrow$ $\exists$ Hilbert space H', $H \subset H'$, and a Riesz basis $\Psi = \{\psi_j\}_{j=1}^{\infty}$ in H', $\forall j$, $\varphi_j = \pi_{H' \to H}(\psi_j)$. (Here π is a orthogonal projection operator.)

Definition For any frame $\phi = \{\varphi_j\}_{j=1}^{\infty}$ $\exists!$ system $\widetilde{\phi} = \{\widetilde{\varphi}_j\}_{j=1}^{\infty}$:

$$\forall f \in H, f = \sum_{j=1}^{\infty}(f, \widetilde{\varphi}_j)\varphi_j. \qquad (*)$$

System $\widetilde{\phi}$ is a frame. It is called the dual frame for ϕ. Expression $(*)$ is called canonical representation of f with respect to ϕ.

$$\sigma_n(f, \phi, H) := \inf_{\Lambda \subset \mathbb{N}, \sharp\Lambda \leq n} \|f - \sum_{j \in \Lambda}(f, \widetilde{\varphi_j})\varphi_j\|_H.$$

$$\sigma_n(K, \phi, H) := \sup_{f \in K} \sigma_n(f, \phi, H).$$

Let us consider the set $\chi = \{\chi_t\}_{t \in [0,a]}$,

$$\chi_t(x) = \chi_{\Omega_t}(x) = \begin{cases} 1, & x \in \Omega_t \\ 0, & x \notin \Omega_t \end{cases} \qquad (**)$$

μ_d is a Lebesque measure and $\mu_d\{\Omega_t\} = t$, $\Omega_{t_1} \subset \Omega_{t_2}$, if $0 < t_1 < t_2 \leq a$.

Theorem $\phi = \{\varphi_j\}_{j=1}^{\infty} \subset L^2(I^d)$, is a frame with boudaries A and B. Then the canonic approximation of class $(**)$ is estimated as follows:

$$\sigma_n(\chi, \phi, L^2(I^d)) \geq C^{-n}, \quad C = C(A, B, a) > 0.$$

(This estimate is exact for Haar o.n.s. we have $\sigma_n(\chi, \phi_0, L^2(0,1)) \le C2^{-n/2}$.)

Theorem $\phi = \{\varphi_j\}_{j=1}^{\infty}$ is a Riesz basis in $L^2(I^d)$, $\|\varphi_j\|_{L^\infty(I^d)} \le M$, $\|\widetilde{\varphi_j}\|_{L^\infty(I^d)} \le M$, $\forall j$. Then for the set $(**)$ we have

$$\sigma_n(\chi, \phi, L^2(I^d)) \ge \frac{C}{\sqrt{n}}, \quad C = C(\phi, a).$$

(This estimate is exact: same estimate from above holds for trigonometric system.)

Multivariate Lagrange Interpolation and an Application of Cayley-Bacharach Theorem

Xue-Zhang Liang[1)], Li-Hong Cui, Jie-Lin Zhang, Ming Zhang

Institute of Mathematics, Jilin University,
Changchun, 130012, P. R. China
1) Email: xzliang@email.jlu.edu.cn

In this paper,we deeply research Lagrange interpolation for the functions of n-variables and give an application of Cayley-Bacharach theorem for it. First we give the concept of the properly posed set of nodes for interpolation in polynomial space $P_m^{(n)}$ (where $P_m^{(n)}$ denotes the space of n-variate polynomials of degree $\leq m$). In practice, the interpolation along algebraic hypersurface or algebraic manifold often be considered. We pose the concept of sufficient intersection about $s(1 \leq s \leq n)$ algebraic hypersurfaces in n-dimensional space and we discuss the interpolation along the algebraic manifold which are sufficient intersection. By means of some theorems (such as Bezout theorem, Macaulay theorem and so on) we prove the dimension of polynomial space $P_m^{(n)}$ along the algebraic manifold $S = s(f_1, \cdots, f_s)$ (where $f_1(X) = 0, \cdots, f_s(X) = 0$ denote s algebraic hypersurfaces) which are sufficient intersection, then give an expression which is convenient for dimension calculation using the forward difference operator. Using Mysovskikh theorem, we give the proofs of the existence of properly posed set of nodes of arbitrary degree for interpolation along 0-dimension and 1-dimension algebraic manifold. Then the existence of properly posed set of nodes of arbitrary degree for interpolation along a algebraic manifold which are sufficient intersection is given. And we give the characterizing conditions of properly posed set of nodes for interpolation. We deduce a general method of constructing properly posed set of nodes for Lagrange interpolation along a algebraic manifold, namely the superposition interpolation process. Then we point out that for s algebraic hypersurface $f_1(X) = 0, \cdots, f_s(X) = 0$ which are sufficient intersection, the set of polynomials $f_1, \cdots, f_s$ must constitute the H-base of ideal $I_s =< f_1, \cdots, f_s >$. At the end of paper, we use the extended case of Cayley-Bacharach theorem to resolve some problems of Lagrange interpolation along 0-dimension and 1-dimension algebraic manifold and find some useful results for practice.

Keywords: Multivariate Lagrange interpolation, Lagrange interpolation along an algebraic manifold, properly posed set of nodes for Lagrange interpolation, Cayley-Bacharach theorem.

On the Degree of Approximation by Spherical Translations

Sheng Baohuai

Department of Mathematics, Shaoxing College of Arts and Sciences,
Shaoxing, Zhejiang, 312000, China
Email: bhsheng@zscas.edu.cn; shengbaohuai@hotmail.com

The degree of approximation of spherical functions by the generalized translation class formed by the generalized operators

$$S_\gamma(f,\ \mu) = \frac{1}{\omega_{q-1}\ \sin^{q-1}\gamma} \int_{\mu\cdot v=\cos\gamma} f(v)\ dv$$

is investigated. The results obtained in the present paper actually imply that the spherical translation class shares the same degree of approximation as that of spherical harmonics.

Approximation to Function $|x|^\alpha$ by Interpolating Polynomials

Wu Xiao-hong[1)] , Lu Zhi-kang[2)]

1) HangZhou WanXiang Polytechnic, HangZhou 310000, China

2) Department of Mathematics,Hangzhou Teacher's College, HangZhou, 310000, China

In 1938 Berstein established the following result: let c_j be real for all j, $n \in N$ and denote $E_n(\alpha) = \min_{c_j} \max_{-1\le x\le 1} ||x|^\alpha - (c_0 + c_1 x + ... + c_n x^n)|$. Then we have the following: $\forall \alpha > 0$, $\forall n \in N$, n even, $n \ge n_0(\alpha)$, we have

$$\frac{C_\alpha}{n^\alpha}(\frac{\pi}{\pi+4})^\alpha \frac{1}{2\sqrt{2}} \le E_n(\alpha) \le \frac{C_\alpha}{n^\alpha},$$

with $C_\alpha = \frac{4}{\pi}|\sin \pi\frac{\alpha}{2}| \int_0^\infty \frac{u^{\alpha-1}}{e^u+e^{-u}} du$.

But, Berstein didn't give a simple construction of those approximating polynomials. Recently, Revers has given a simple construction of those approximating polynomials at the case of $\alpha \in (0, 2/3]$. Lately, HE Guo-long canceled the restriction of α, proved Revers' theorem also successed in $\alpha \in \left(\frac{2}{3}, 1\right]$.

In our paper, we discuss those constructions of interpolating polynomials to best approximate function $|x|^\alpha$ at the case of $0 < \alpha < 2$. It is showed in the following:

1. Let $n = 2m$, $m \in N$, $0 < \alpha \le 1$, then we have $F_n(\alpha) < \dfrac{1}{(n+1)^\alpha}$, where $F_{2m}(\alpha) = \max_{-1\le x\le 1} ||x|^\alpha - R_{2m}(x)|$, in which $R_{2m}(x)$ is the Lagrange interpolating polynomial to $|x|^\alpha$ on $[-1, 1]$ at the Chebyshev nodes: $x_j = \cos(j - \frac{1}{2})\frac{\pi}{2m+1} (j = 1, 2, ..., 2m + 1)$

2. Let $n = 2m$, $m \in N$, $1 < \alpha < 2$, then we have $F_n(\alpha) < \dfrac{C_{\alpha,n}}{n^\alpha}$, and $\lim_{n\to\infty} C_{\alpha,n} = \pi(\alpha + 3) + (\frac{\pi}{2})^{\alpha-1}$. where $F_{2m}(\alpha) = \max_{-1\le x\le 1} ||x|^\alpha - R_{2m}(x)|$, in which $R_{2m}(x)$ is the Lagrange interpolating polynomial to $|x|^\alpha$ on $[-1, 1]$ at the Chebyshev nodes: $x_0 = 0$, $x_j = \cos(j - \frac{1}{2})\frac{\pi}{2m} (j = 1, 2, ..., 2m)$.

3.Let $n = 2m$, $m \in N$, $0 < \alpha \le 1$, then we have $F_n(\alpha) < \dfrac{C_\alpha}{(n+2)^\alpha}$, where $F_{2m}(\alpha) = \max_{-1\le x\le 1} ||x|^\alpha - Q_{2m}(x)|$, in which $Q_{2m}(x)$ is the Lagrange interpolating polynomial to $|x|^\alpha$ on $[-1, 1]$ at the Chebyshev nodes: $x_j = \cos\frac{j\pi}{2m+2} (j = 1, 2, ..., 2m + 1)$, and C_α is relevant to α.

Using Splines to Study Some Problems in Combinatorics: A Survey

Zhiqiang Xu

Institute of Computational Mathematics,
Academy of Mathematics and System Sciences,
Chinese Academy of Sciences, Beijing, China

In this talk, we shall survey these results as follows about solving problems in combinatorics using multivariate splines.

1. By using multivariate splines, we shall give an explicit formulation for counting non-negative solutions for linear Diophantine equations.
2. We shall show the iterated formulation for computing multivariate cone splines can be used to compute the volume of convex polytopes.
3. According to multivariate Box splines, the famous Popoviciu's formulation in number theory is generalized.
4. An explicit formulation for Ehrhart polynomial which count integer points in polytopes is also given.

Finally, some challenging problems in this area are also raised.

Convergence Properties of Control Meshes of Catmull-Clark Subdivision Surfaces

Xiao-Ming Zeng

Department of Mathematics, Xiamen University, Xiamen 361005, China

Subdivision surfaces are powerful and useful technique in modeling free-form surfaces. They can be widely used in many fields, such as computer-aided design, graphical modeling, computer animation, medical image processing and so on.

The Catmull-Clark subdivision surface was designed to generalize the bi-cubic B-spline surface to the meshes of arbitrary topology. In Catmull-Clark scheme, an initial control mesh is refined by adding new vertices, faces and edges at each subdivision step. In the limit as the number of subdivision steps goes to infinity, the control mesh converges to the Catmull-Clark limit surface. Thus Catmull-Clark scheme is an approximation subdivision scheme. A computational formula of depth for Catmull-Clark subdivision surfaces was presented in [Journal of Computational and Applied Mathematics, available online 6 September 2005]. In this paper by the concept of neighbor points and using the second difference of control points, we obtain a rate of convergence of control meshes of Catmull-Clark surface. Furthermore, by means of this result of convergence we derive a new computational formula of subdivision depth for Catmull-Clark surface.

The Complete Asymptotic Expansion for the Baskakov Operators[3]

Chun-gou Zhang[1)], Quan-e Wang

Department of Mathematics, Capital Normal University
Beijing 100037, People's Republic of China
1) Email: zhangcg@mail.cnu.edu.cn

In this paper, we derive the clasical Baskakov operators $V_n(f;x)$ the complete asymptotic expansion in the form of all coefficients of n^{-k}, $k = 0, 1 \cdots$ being calculated explicitly in terms of Stirling number of the first and secong kind and another number $G(i,p)$. As a corollary, we also get the Voronovskaja-type result for the operators.

Keywords:the Baskakov operators, the Meyer-König and Zeller operators, complete asymptotic expansion, Stirling numbers.

[3]Supported by the Natural Science Foundation of China (10371080)

Opening Speech

by Professor Lizhi Xu (Hsu, L.C.)

23rd April, 2006, Kunming, China

I am very glad to have this chance to deliver an opening speech for the International Symposium on Approximation Theory and Remote Sensing Applications.

First of all, as a honorary chairman of this conference, let me express our warm welcome to all of you, participating in this International Conference. In particular, we feel very happy and much honored by the presentation of several world renowned mathematicians, namely Brown, Ditzian, Kashin, Pereverzyev and Pinkus, and some others.

We are also very much delighted to see that more than half of participants are younger scholars and young mathematicians who are doing either pure theoretical researches or practical applications.

As you know, Approximation Theory is one of the most favorite branch of mathematical analysis that has been developing in China for almost 50 years. I have the real pleasure to mention a historical fact that some early Russian text books and mathematics literature had brought deep influence on the mathematics teaching and research in various key universities of China during 50's through 60's of the last century. For instance, Natanson's textbook "Constructive Theory of Functions", Gontscharoff's treatise "Interpolation and Approximation", Korovkin's booklet on "the linear positive operators" as well as various classical papers by S. Bernstein, A. N. Kolmogoroff, and many others have inspired and introduced Chinese scholars and students to enter into the fascinating field of Approximation Theory and related branches. Consequently, as encouraged and supported by Chinese Math. Society and Chinese learned mathematicians, there had been various mathematical seminars and workshops on Approximation Theory (Constructive Function Theory) that took places in several key universities of China in the last century. However, such happy academic activities were unfortunately interrupted during the trying years of the so-called cultural revolution of China. Chinese mathematicians then couldn't touch current mathematics literature for at least 10 years. Then there came the "Spring Season of Sciences" in China, roughly, the years started from 1980 onwards. Henceforth, Chinese math. Analysts and their students not only began to restore their interests in Approximation Theory, but also got chance to give consideration in connection with some new branches and applications. As a result, a number of graduate textbooks and reference books related to Approximation Theory written by Chinese researchers in either English or Chinese have been published during recent years. Also, you may find that during recent

years more and more Chinese younger scholars and researchers who got some basic knowledge and training in Approximation Theory could publish various papers in some related fields such as Numerical Approximations, Computational Analysis, Harmonic Analysis, Wavelets, Fractals, Function Spaces Learning Theory, Artificial Intelligence, and so forth.

Actually, the research tendency described above may be of some general characteristic, and has encouraged the change of name for an international journey called "Approximation Theory and Its Applications" (ATA) and published in China into the new name "Analysis in Theory and Applications", still retaining the original abbreviation "ATA". The ATA journal was established and started publication 20 years ago, having honorary editors G.G. Lorentz and S.M.Nikolskii, and financially supported by 7 Chinese key universities. And the new name for the journal was proposed by P.L. Butzer. I think, the change of name for our ATA journal just means that Approximation Theory as a mathematical tool has greatly extended its range of applications during recent 20 years! An immediate example is just given by the present conference: One may ask why we call the present conference as Symposium on Approximation Theory and Remote Sensing Applications! Obviously the reason is simply that Approximation Theory has been also found effective applications to the Remote Sensing Techniques.

Certainly, it is our sincere hope that this symposium will provide for mathematicians some opportunities for exchange of views and ideas regarding scientific researches or some open problems await for resolutions.

Finally let me sincerely wish that all of you, the participants of this conference will enjoy your nice time of visits in this so-called "Spring City of China"-Kunming.

Thank you for your attendance!

Closing Speech

by Professor Tang Ping

24th April, 2006, Kunming, China

Honorable Mr. President, Ladies and Gentlemen:

Good afternoon. Thanks to the joint efforts and contribution from all participants, the International Symposium on Approximation Theory and Remote Sensing Applications is going to be closed successfully.

In the past two days, we have listened to the 11 keynote speaker's presentations and made discussions in the plenary session. We thank our eminent speakers for sharing and exchanging their views and ideas regarding scientific researches and applications with us. And we write down some special impressions to share with you.

Firstly, classical approximation problems are still paid much attentions. In this direction, Brown discussed positivity and boundedness of trigonometric sums. Allan Pinkus discussed the density methods in approximation theory. Xie Tingfan surveyed rational approximation of function , and Zhou Songping investigated some conditions in Fourier analysis.

Secondly, the research on spherical harmonic analysis becomes more and more active in recent years. It is concluded from Wang Kunyang's review of his group's work after 2001 and Ditzian's talk, who discussed the various measures of smoothness for different spaces of functions on the sphere.

Thirdly, some new approximation theories from applications, such as M-term approximation and learning theory, have already made progresses and applied to the applications. Kashin in his talk presented the estimates of best M-term approximation and the result about properties of random matrices in image processing. Peng Lizhong discussed wavelets and DCT transforms performance for image /video compression. Pereverzyev tried to derive a better learning algorithms based on discussion a relation between learning theory and regularization of linear ill-posed inverse problem.

Fourthly, approximation theory has indeed greatly extended its range of researches and applications. It also includes various approximation algorithms. Xu Shuzhan from industry aims to get the approximation algorithm to solve a kind of linear system with high dimensions. Dr. Wang Yanfei investigated the norm approximation solution method by regularization to retrieve a kind of land surface parameters in remote sensing inverse problem. We believe that in the future communications between applications and approx-

imation theory will become a continual work.

Finally, on behalf of the organizing committee, I thank the National Natural Science Foundation, Beijing Normal University, Institute of Remote Sensing Applications (CAS), Yunnan Nationalities University for their financial sponsoring and strong support. And I thank everyone who worked for our conference and made it to be a success. Now, I declare the closing of the International Symposium on Approximation Theory and Remote Sensing Applications.

Thank you.

A notice to all participants

The journal ATA (Analysis in Theory and Applications) would like to publish the papers presented on the Conference. Please send your papers which were presented as plenary reports or posters to ATA in appropriate form required by the journal for their consideration of publication.

Attachment The group photo of all participants