An Example

Let

$$f(x) = x^2 \sin x^{-2} \text{ for } x \neq 0, \text{ and } f(0) = 0.$$

Then

$$f'(x) = 2x \sin x^{-2} - 2x^{-1} \cos x^{-2} \text{ for } x \neq 0, \text{ and } f'(0) = 0.$$

We see that $f' : \mathbb{R} \longrightarrow \mathbb{R}$ is discontinuous at $x = 0$.