



## 2023年北京师范大学偏微分方程数值解研讨会(线上)

中国 北京

2023年4月22-23日

主办单位：北京大学数学科学学院

## 2023年北京师范大学偏微分方程数值解研讨会(线上)

日期: 2023年4月22-23日

主办方: 北京师范大学数学科学学院

会议地点(Zoom线上);

会议号: 897 335 27812 密码: 689713 (4月22日)

会议号: 882 426 00880 密码: 768254 (4月23日)

### 学术委员会:

舒其望 (美国布朗大学)

张智民 (美国韦恩州立大学)

### 组委会:

曹外香 (北京师范大学)

### 报告人(按姓氏字母排序):

邓伟华 (兰州大学)

金石 (上海交通大学)

明平兵 (中国科学院数学与系统科学研究院)

沈捷 (美国普渡大学)

舒其望 (美国布朗大学)

武海军 (南京大学)

许学军 (同济大学)

张磊 (北京大学)

周涛 (中国科学院数学与系统科学研究院)

### 会议资助:

北京师范大学数学科学学院2023年双一流学科经费

国家自然科学基金

### 会务联系人:

曹外香 18701390316 caowx@bnu.edu.cn

## 会议日程

2023 年 4 月 22 日		
Zoom ID: 897 335 27812 Password: 689713		
北京时间	报告题目	报告人
8:00-8:10	开幕式	张智民
主持人: 张智民		
8:10-9:00	Inverse Lax-Wendroff Procedure for Numerical Boundary Conditions	舒其望
主持人: 舒其望		
9:00-9:50	Adaptive FEM for Helmholtz equation with large wave number	武海军
9:50-10:40	构造复杂系统的解景观	张磊
午休		
主持人: 曹外香		
15:00-15:50	Quantum Computation of partial differential equations	金石
15:50-16:40	非静止介质中反常与非遍历多尺度模型及算法	邓伟华
16:40-17:30	Deep adaptive sampling for numerical PDEs	周涛

2023 年 4 月 23 日		
Zoom ID: 882 426 00880 Password: 768254		
北京时间	报告题目	报告人
主持人: 张智民		
8:00-8:50	Generalized SAV approach for general dissipative nonlinear systems with particular application to Navier-Stokes equations	沈捷
主持人: 蔡勇勇		
8:50-9:40	Domain Decomposition Learning Methods	许学军
9:40-10:30	Large plate bending with isometry constraint: FEM and machine learning	明平兵

## 学术报告题目和摘要

### 1. 报告人: 舒其望 (美国布朗大学)

题目: Inverse Lax-Wendroff Procedure for Numerical Boundary Conditions

摘要: When solving partial differential equations, finite difference methods have the advantage of simplicity, however they are usually only designed on Cartesian meshes. In this talk, we will discuss a class of high order finite difference numerical boundary condition for solving hyperbolic Hamilton-Jacobi equations, hyperbolic conservation laws, and convection-diffusion equations on complex geometry using a Cartesian mesh. The challenge results from the wide stencil of the interior high order scheme and the fact that the boundary may not be aligned with the mesh. Our method is based on an inverse Lax-Wendroff procedure for the inflow boundary conditions coupled with traditional extrapolation or weighted essentially non-oscillatory (WENO) extrapolation for outflow boundary conditions. The schemes are shown to be high order and stable, under the standard CFL condition for the inner schemes, regardless of the distance of the first grid point to the physical boundary, that is, the "cut-cell" difficulty is overcome by this procedure. Recent progress in nonlinear conservation laws with sonic points, and a conservative version of the method, will be discussed. Numerical examples are provided to illustrate the good performance of our method.

### 2. 报告人: 武海军 (南京大学)

题目: Adaptive FEM for Helmholtz equation with large wave number

摘要: A posteriori upper and lower bounds are derived for the finite element method (FEM) for the Helmholtz equation with large wavenumber. It is proved rigorously that the standard residual type error estimator seriously underestimates the true error of the FE solution for the mesh size  $h$  in the preasymptotic regime, which is first observed by [Babuska, et al., A posteriori error estimation for finite element solutions of Helmholtz equation. Part I, Int. J. Numer. Meth. Engrg. 40, 3443--3462 (1997)] for a one dimensional problem. By establishing an equivalence relationship between the error estimators for the FE solution and the corresponding elliptic projection of the exact solution, an adaptive algorithm is proposed and its convergence and quasi-optimality are proved under the condition that  $\$k^{2p+1}h_{0}^{2p}\$$  is sufficiently small, where  $k$  is the wavenumber,  $\$h_0\$$  is the initial mesh size.

### 3. 报告人: 张磊 (北京大学)

题目: 构造复杂系统的解景观

摘要: 很多实际的应用问题都可以被归为计算数学中求解具有多个变量的非线性能量函数的极小值问题。这类多解问题通常具有多个极小, 那么如何寻找全局极小和如何找到不同极小之间的关系一直是计算数学领域的两个关键问题。在报告中, 我们提出了一个新的解景观

(solution landscape) 概念。解景观描述了不同的极小被相应的一阶鞍点连接, 低阶鞍点被相应的高阶鞍点连接, 最终连接到一个最高阶鞍点的层次结构图。我们根据解景观的特征, 利用发展的鞍点动力学, 结合向下搜索和向上搜索方法, 可以高效地构建出完整的解景观。我们以软物质系统中的液晶和准晶为例, 系统地构建了向列相液晶的缺陷景观以及发现了从晶体到准晶的形核过程。

4. 报告人: ( 大学)

题目: Quantum Computation of partial differential equations

摘要: Quantum computers have the potential to gain algebraic and even up to exponential speed up compared with its classical counterparts, and can lead to technology revolution in the 21st century. Since quantum computers are designed based on quantum mechanics principle, they are most suitable to solve the Schrodinger equation, and linear PDEs (and ODEs) evolved by unitary operators. The most efficient quantum PDE solver is quantum simulation based on solving the Schrodinger equation. It became challenging for general PDEs, more so for nonlinear ones. Our talk will cover three topics:

1) We introduce the "warped phase transform" to map general linear PDEs and ODEs to Schrodinger equation or with unitary evolution operators in higher dimension so they are suitable for quantum simulation;

2) For (nonlinear) Hamilton-Jacobi equation and scalar nonlinear hyperbolic equations we use the level set method to map them—exactly—to phase space linear PDEs so they can be implemented with quantum algorithms and we gain quantum advantages for various physical and numerical parameters.

For PDEs with uncertain coefficients, we introduce a transformation so the uncertainty only appears in the initial data, allowing us to compute ensemble averages with multiple initial data with just one run, instead of multiple runs as in Monte-Carlo or stochastic collocation type sampling algorithms.

3. 报告人: ( 大学)

题目: Quantum Computation of partial differential equations

摘要: Quantum computers have the potential to gain algebraic and even up to exponential speed up compared with its classical counterparts, and can lead to technology revolution in the 21st century. Since quantum computers are designed based on quantum mechanics principle, they are most suitable to solve the Schrodinger equation, and linear PDEs (and ODEs) evolved by unitary operators. The most efficient quantum PDE solver is quantum simulation based on solving the Schrodinger equation. It became challenging for general PDEs, more so for nonlinear ones. Our talk will cover three topics:

1) We introduce the "warped phase transform" to map general linear PDEs and ODEs to Schrodinger equation or with unitary evolution operators in higher dimension so they are suitable for quantum simulation;

2) For (nonlinear) Hamilton-Jacobi equation and scalar nonlinear hyperbolic equations we use the level set method to map them—exactly—to phase space linear PDEs so they can be implemented with quantum algorithms and we gain quantum advantages for various physical and numerical parameters.

For PDEs with uncertain coefficients, we introduce a transformation so the uncertainty only appears in the initial data, allowing us to compute ensemble averages with multiple initial data with just one run, instead of multiple runs as in Monte-Carlo or stochastic collocation type sampling algorithms.

