2023年北京师范大学偏微分方程数值解研讨会(线上)

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2023 年 4 月 22-23 日

主办单位：北京师范大学数学科学学院

2023年北京师范大学偏微分方程数值解研讨会(线上)

日期: 2023年4月22-23日
主办方: 北京师范大学数学科学学院
会议地点(Zoom线上);
会议号: 897 335 27812　密码: 689713 (4月22日)
会议号: 882 426 00880　密码: 768254 (4月23日)

学术委员会:
舒其望　　　(美国布朗大学)
张智民　　　(美国韦恩州立大学)

组委会:
曹外香　　　(北京师范大学)

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邓伟华　　　(兰州大学)
金石　　　　(上海交通大学)
明平兵　　　(中国科学院数学与系统科学研究院)
沈捷　　　　(美国普渡大学)
舒其望　　　(美国布朗大学)
武海军　　　(南京大学)
许学军　　　(同济大学)
张磊　　　　(北京大学)
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会议日程

2023 年 4 月 22 日		
Zoom ID: 897 335 27812　Password: 689713		
北京时间	报告题目	报告人
8:00-8:10	开幕式	张智民
主持人: 张智民		
8:10-9:00	Inverse Lax-Wendroff Procedure for Numerical Boundary Conditions	舒其望
主持人: 舒其望		
9:00-9:50	Adaptive FEM for Helmholtz equation with large wave number	武海军
9:50-10:40	构造复杂系统的解景观	张磊
午休		
主持人: 曹外香		
14:30-15:20	Quantum Computation of partial differential equations	金石
15:20-16:10	Deep adaptive sampling for numerical PDEs	周涛
16:10-17:00	Large plate bending with isometry constraint: FEM and machine learning	明平兵

2023 年 4 月 23 日		
Zoom ID: 882 426 00880　Password: 768254		
北京时间	报告题目	报告人
主持人: 张智民		
8:00-8:50	Generalized SAV approach for general dissipative nonlinear systems with particular application to Navier-Stokes equations	沈捷
主持人: 蔡勇勇		
8:50-9:40	Domain Decomposition Learning Methods	许学军
9:40-10:30	非静止介质中反常与非遍历多尺度模型及算法	邓伟华

学术报告题目和摘要

1. 报告人: 舒其望 （美国布朗大学）

题目: Inverse Lax-Wendroff Procedure for Numerical Boundary Conditions

摘要: When solving partial differential equations, finite difference methods have the advantage of simplicity, however they are usually only designed on Cartesian meshes. In this talk, we will discuss a class of high order finite difference numerical boundary condition for solving hyperbolic Hamilton-Jacobi equations, hyperbolic conservation laws, and convection-diffusion equations on complex geometry using a Cartesian mesh. The challenge results from the wide stencil of the interior high order scheme and the fact that the boundary may not be aligned with the mesh. Our method is based on an inverse Lax-Wendroff procedure for the inflow boundary conditions coupled with traditional extrapolation or weighted essentially non-oscillatory (WENO) extrapolation for outflow boundary conditions. The schemes are shown to be high order and stable, under the standard CFL condition for the inner schemes, regardless of the distance of the first grid point to the physical boundary, that is, the "cut-cell" difficulty is overcome by this procedure. Recent progress in nonlinear conservation laws with sonic points, and a conservative version of the method, will be discussed. Numerical examples are provided to illustrate the good performance of our method.

2. 报告人: 武海军 （南京大学）

题目: Adaptive FEM for Helmholtz equation with large wave number

摘要: A posteriori upper and lower bounds are derived for the finite element method (FEM) for the Helmholtz equation with large wavenumber. It is proved rigorously that the standard residual type error estimator seriously underestimates the true error of the FE solution for the mesh size h in the preasymptotic regime, which is first observed by [Babuska,~et~al., A posteriori error estimation for finite element solutions of Helmholtz equation. Part I, Int. J. Numer. Meth. Engrg. 40, 3443--3462 (1997)] for a one dimensional problem. By establishing an equivalence relationship between the error estimators for the FE solution and the corresponding elliptic projection of the exact solution, an adaptive algorithm is proposed and its convergence and quasi-optimality are proved under the condition that $k^{2p+1}h_0^{2p}$ is sufficiently small, where k is the wavenumber, h_0 is the initial mesh size.

3. 报告人: 张磊（北京大学）

题目: 构造复杂系统的解景观

摘要: 很多实际的应用问题都可以被归为计算数学中求解具有多个变量的非线性能量函数的极小值问题。这类多解问题通常具有多个极小，那么如何寻找全局极小和如何找到不同极小之间的关系一直是计算数学领域的两个关键问题。在报告中，我们提出了一个新的解景观(solution landscape) 概念。解景观描述了不同的极小被相应的一阶鞍点连接，低阶鞍点被相应的高阶鞍点连接，最终连接到一个最高阶鞍点的层次结构图。我们根据解景观的特征，利用发展的鞍点动力学，结合向下搜索和向上搜索方法，可以高效地构建出完整的解景观。我们以软物质系统中的液晶和准晶为例，系统地构建了向列相液晶的缺陷景观以及发现了从晶体到准晶的形核过程。

4.报告人: 金石 （上海交通大学）

题目: Quantum Computation of partial differential equations

摘要: Quantum computers have the potential to gain algebraic and even up to exponential speed up compared with its classical counterparts, and can lead to technology revolution in the 21st century. Since quantum computers are designed based on quantum mechanics principle, they are most suitable to solve the Schrodinger equation, and linear PDEs (and ODEs) evolved by unitary operators. The most efficient quantum PDE solver is quantum simulation based on solving the Schrodinger equation. It became challenging for general PDEs, more so for nonlinear ones. Our talk will cover three topics:

1)We introduce the "warped phase transform" to map general linear PDEs and ODEs to Schrodinger equation or with unitary evolution operators in higher dimension so they are suitable for quantum simulation;

2)For (nonlinear) Hamilton-Jacobi equation and scalar nonlinear hyperbolic equations we use the level set method to map them—exactly—to phase space linear PDEs so they can be implemented with quantum algorithms and we gain quantum advantages for various physical and numerical parameters.

For PDEs with uncertain coefficients, we introduce a transformation so the uncertainty only appears in the initial data, allowing us to compute ensemble averages with multiple initial data with just one run, instead of multiple runs as in Monte-Carlo or stochastic collocation type sampling algorithms.

5.报告人: 周涛（中国科学院数学与系统科学研究院）

题目: Deep adaptive sampling for numerical PDEs

摘要: Adaptive computation is of great importance in numerical simulations. The ideas for adaptive computations can be dated back to adaptive finite element methods in 1970s. In this talk, we shall first review some recent development for adaptive method with applications. Then, we shall propose a deep adaptive sampling method for solving PDEs where deep neural networks are utilized to approximate the solutions. In particular, we propose the failure informed PINNs (FI-PINNs), which can adaptively refine the training set with the goal of reducing the failure probability. Compared to the neural network approximation obtained with uniformly distributed collocation points, the developed algorithms can significantly improve the accuracy, especially for low regularity and high-dimensional problems.

6.报告人: 明平兵 （中国科学院数学与系统科学研究院）

题目: Large plate bending with isometry constraint: FEM and machine learning

摘要: We study a minimization problem arising from a plate bending model with isometry constraints. Both finite element method and machine learning oriented method are exploited to investigate the convergence of the minimizer, and to what degree the isometry constraints are kept by the proposed methods. We found that FEM may have certain a-priori theoretical convergence, while machine learning method seems more efficient in most cases under study. This is a joint work with Xiang Li and Yulei Liao (AMSS, CAS).

7. 报告人: 沈捷（美国普渡大学）

题目: Generalized SAV approach for general dissipative nonlinear systems with particular application to Navier-Stokes equations

摘要: I will present some recent advances on using the generalized scalar auxiliary variable (GSAV) approach to develop highly efficient and accurate schemes for a large class of dissipative nonlinear systems. These schemes are easy to implement, can be high-order in time, and lead to a uniformly bound for the numerical solution in a suitable norm that plays an essential role in corresponding error analysis. It can also be positivity/bound preserving through a function transform.

We combine this GSAV approach with a new consistent splitting scheme for the Navier-Stokes equations, and obtain, for the first time, a strongly unconditional stable, decoupled schemes with uniform second-order accuracy for the velocity and pressure.

8. 报告人: 许学军（同济大学）

题目: Domain Decomposition Learning Methods

摘要: With the aid of hardware and software developments, there has been a surge of interests in solving partial differential equations by deep learning techniques, and the integration with domain decomposition strategies has recently attracted considerable attention due to its enhanced representation and parallelization capacity of the network solution. In this talk, a novel learning approach, i.e., the compensated deep Ritz method, is proposed to enable the flux transmission across subregion interfaces with guaranteed accuracy, thereby allowing us to construct effective learning algorithms for realizing the more general non-overlapping domain decomposition methods in the presence of overfitted interface conditions. Numerical experiments on a series of elliptic boundary value problems including the regular and irregular interfaces, low and high dimensions, smooth and high-contrast coefficients on multidomains are carried out to validate the effectiveness of our proposed domain decomposition learning algorithms.

9. 报告人: 邓伟华（兰州大学）

题目: 非静止介质中反常与非遍历多尺度模型及算法

摘要: 在静止介质中的反常与非遍历多尺度动力学的研究框架已经建立, 包括模型、分析、算法和应用, 尽管还有重要的有待攻克的难题、各种新观测到的现象还不断涌现. 更多的粒子动力学发生在非静止介质中, 尤其在生命科学和宇宙学中, 在我们的日常生活中也同样普遍. 流体是最具代表性的非静止介质. Navier-Stokes方程、Boltzmann方程、Navier Stokes、Euler方程等是其中的主要宏观方程, 还有一些微观的动理学方程. 我们将探讨相关的模型和算法.