Conference on Representation Theory of Algebras

In honor of the 80th birthday of Professor Claus Michael Ringel

Beijing Normal University

May 29–June 2, 2025

Program and Abstracts

Organizing committee:

Bangming Deng, Jinyun Guo, Yanan Lin, Liangang Peng, Changchang Xi, Jie Xiao, Pu Zhang, Yingbo Zhang, Bin Zhu

Accommodation: Holiday Inn Beijing Deshengmen: No. 71, Deshengmenwai Street, Xicheng District, Beijing (北京德胜门华宇假日酒店: 北京西城区德胜门外大街71号)

Conference Venue: Jinshixuetang (京师学堂) at Beijing Normal University

Program

Friday 30 May	
08:30-09:00	Registration and Opening
09:00-09:50	**Gordana Todorov**, Northeastern University *TBA*
10:00-10:50	**Yu Zhou**, Beijing Normal University *τ-tilting theory in extended module categories*
10:50-11:10	Tea break
11:10-12:00	**Fan Qin**, Beijing Normal University *Cluster algebras, derived Hall algebras, and braid group actions*
12:00-14:00	Lunch at Holiday Inn（北京德胜门华宇假日酒店）
14:30-15:20	**Yu Qiu**, Tsinghua University *From green mutation to X-evolution on cluster complexes*
15:20-15:40	Tea break
15:40-16:30	**Hongxing Chen**, Capital Normal University *Some advances and prospect on the Nakayama conjecture*
16:40-17:30	**Markus Schmidmeier,** Florida Atlantic University *Invariant subspaces of nilpotent operators. The power of tubular algebras*
18:00-	Dinner at Holiday Inn（北京德胜门华宇假日酒店）
Saturday 31 May	
09:00-09:50	**Sibylle Schroll,** Universität zu Köln *On algebras derived equivalent to skew-gentle algebras*
10:00-10:50	**Ming Lu,** Sichuan University *Semi-derived Ringel-Hall algebras, quantum groups and i-quantum groups*
10:50-11:10	Tea break
11:10-12:00	**Nan Gao**, Shanghai University *Claus Michael Ringel's main contributions to Gorenstein-projective modules*
12:00-14:00	Lunch at Holiday Inn（北京德胜门华宇假日酒店）
14:30-15:20	**Aslak Bakke Buan,** Norwegian University of Science and Technology *On a generalization of exceptional sequences*
15:20-15:40	Tea break
15:40-15:55	**Gabriella D'Este,** Università di Milano *A theorem on support τ-tilting pairs*
16:00-16:15	**Buyan Li**, Tsinghua University *Notes on Chevalley Groups and Root Category*

16:20-16:35	**Jianmin Chen**, Xiamen University *Tilting objects on Geigle-Lenzing projective spaces*
16:40-16:55	**Jie Xiao**, Beijing Normal University *The parity of Lusztig's restriction functor and Green's formula*
18:00--	Conference Dinner at Holiday Inn（北京德胜门华宇假日酒店）
Sunday 1 June	
09:00-09:50	**Naihuan Jing**, North Carolina State University *q-Immanants and higher quantum Capelli identities*
10:00-10:50	**Rasool Hafezi,** Nanjing University of Information Science and Technology *Representation theory of monomorphism categories*
10:50-11:10	Tea break
11:10-12:00	**Xiao-Wu Chen**, University of Science and Technology of China *Algebras and categories associated to Cartan matrices*
12:00-14:00	Lunch at Holiday Inn（北京德胜门华宇假日酒店）
14:30-14:45	**Liangang Peng**, Sichuan University *On modules determined uniquely by their dimension vectors*
14:50-15:05	**Wen Chang**, Shaanxi Normal University *Tilting-completion for gentle algebras*
15:10-15:25	**Peigen Cao**, University of Science and Technology of China *Cluster-additive functions and Ringel's conjecture*
15:25-15:50	Tea break
15:50-16:40	**Shiquan Ruan**, Xiamen University *Ringel-Hall algebra approach to i-quantum groups*
16:50-17:40	**Osamu Iyama**, University of Tokyo *TBA*
18:00-	Dinner at Holiday Inn (北京德胜门华宇假日酒店)
Monday 2 June	
09:00-09:50	**Jie Du**, University of New South Wales *Finite dimensional algebras and Lie theory*
10:00-10:50	**Bernhard Keller**, Université Paris Cité *Gorenstein projective dg modules in cluster theory*
11:30-14:00	Lunch at Holiday Inn（北京德胜门华宇假日酒店）
14:00-	**Free Discussion & Departure**

Lecture on

German Fascination with Chinese Culture in the 18th century

Claus Michael Ringel (Universität Bielefeld)

May 31, 17:00–17:30

Abstract. When I visited the Beijing Kong Miao in 2024, I appreciated very much a presentation about the international reputation of Confucius; in particular, there is a reference to Leibniz (1646-1716). A second German philosopher and scientist deserves to be recalled in this context, namely Christian Wolff (1679-1754). The Leibniz–Wolff philosophy was the main philosophical school in Germany before Kant, with strong emphasis on mathematics and natural sciences, trying to incorporate Newtonian physics into a coherent ontological scheme.

When Wolff in 1721 stepped down as pro-rector of the university of Halle, his official lecture had the title

"The practical religion of the Chinese",

focussing the attention to what Wolff had learned about Confucius. After delivering the lecture, Wolff was expelled from Prussia and could return to Halle only after 19 years! But during all that time: his exil at Marburg, but also his return to Halle, he was highly praised in Germany as well as outside.

Abstract

Aslak Bakke Buan (Norwegian University of Science and Technology)

On a generalization of exceptional sequences

Inspired by τ-tilting theory, the concept of τ-exceptional sequences was introduced in joint work with Marsh. They are sequences of certain indecomposable modules, meeting some homological criteria. For any finite-dimensional algebra, such sequences of length equal to the rank of the algebra exist. For hereditary algebras, they coincide with the classical exceptional sequences. In recent joint work with Hanson and Marsh, a mutation operation on τ-exceptional sequences was introduced.

I will give an overview of this work, including some recent results and progress in joint projects with several authors. In particular, some recent joint work with Kaipel and Terland, and also, some results from Terland's PhD work, will be included.

Peigen Cao (University of Science and Technology of China)

Cluster-additive functions and Ringel's conjecture

Let A be a symmetrizable generalized Cartan matrix of size r. A cluster-additive function associated to A is a map from $[1, r] \times \mathbb{Z}$ to $\mathbb{Z}$ satisfying certain mesh type relations. Such functions were introduced by Ringel in 2012 as cluster version of additive functions in the representation theory. In the finite type case, Ringel conjectured that any cluster-additive function is periodic and can be written as a non-negative linear combination of cluster-hammock functions, which are a class of "elementary cluster-additive functions".

In this talk, we will give some links between cluster-additive functions and cluster theory, which can be used to prove Ringel's conjecture.

Wen Chang (Shaanxi Normal University)

Tilting-completion for gentle algebras

It is proved that any almost tilting module over a gentle algebra is partial tilting, that is, it can be completed as a tilting module. Furthermore, it has at most $2n$ complements, which confirms a (deformed)conjecture of Happel for the case of gentle algebras.

At the same time, for any $n \geq 3$ and $1 \leq m \leq n-2$, there always exists a connected gentle algebra with rank n and a pre-tilting module over it with rank m which is not partial tilting. The tool we use is the surface model associated with the module category of a gentle algebra.

Hongxing Chen (Capital Normal University)

Some advances and prospect on the Nakayama conjecture

In the representation theory of Artin algebras, the long-standing and not yet solved Nakayama conjecture says that a finite-dimensional algebra over a field with infinite dominant dimension is self-injective. This is one of the main homological conjectures in representation theory. In this talk, we introduce some new advances and related open questions on this conjecture. This is based on several joint work with Changchang Xi, and also with Ming Fang.

Jianmin Chen (Xiamen University)

Tilting objects on Geigle–Lenzing projective spaces

In this talk, I will present our recent progress on tilting objects over Geigle–Lenzing projective spaces, include: introducing the notion of 2-(co)extension bundles and applying them to construct tilting objects in the stable categories of arithmetically Cohen–Macaulay bundles on Geigle–Lenzing projective spaces of weight quadruple, constructing a family of 2-tilting bundles for Geigle–Lenzing projective spaces of type $(2, 2, p, q)$, and so forth. This is based on joint work with Shiquan Ruan and Weikang Weng.

Xiao-Wu Chen (University of Science and Technology of China)

Algebras and categories associated to Cartan matrices

It is well known that the representation theory of finite quivers is related to root systems and Weyl groups. For the non-simply-laced cases, one replaces quivers by quivers with automorphisms. In 2017, Geiss–Leclerc–Schroeer introduced a class of 1-Gorenstein algebras associated to Cartan matrices, and used tau-locally-free modules to categorify the root systems of non-simply-laced Dynkin diagrams. We show that in "bad" characteristic, the GLS algebras are Morita equivalent to certain skew group algebras of path algebras. This allows us to compare the GLS treatment with the more classical one. The

key ingredients are EI categories of Cartan type and orbifold quotients of EI quivers. This is based on a series of joint papers with Ren Wang in HUT.

Gaberiella D'Este (Università di Milano)

A theorem on support τ-tilting pairs

I will describe a bijection between the indecomposable summands of two modules of the form $P \oplus T$ and $P' \oplus T'$ such that (T, P) and (T', P') are two basic support τ-tilting pairs in the sense of [1]. The bijection obtained extends the bijections constructed in [2] and [3].

References

[1] Adachi T., Iyama O. and Reiten I., *τ-tilting theory*, Composition Mathematica, 150(3), (2014), 415–452.

[2] D'Este G. and Tekin Akcin H. M., *A bijection between the indecomposable summands of two multiplicity free tilting modules*, Bulletin of the Iranian Mathematical Society, 48 (2022), 2521–2538.

[3] D'Este G. and Tekin Akcin H. M., *Bijections between τ-rigid modules*, to appear in Contemprorary Mathematics, Proceedings of the 14th Ukraine Algebra Conference.

[4] D'Este G., *A theorem on support τ-tilting pairs*, preprint

Jie Du (University of New South Wales)

Finite dimensional algebras and Lie theory

The Ringel–Hall algebra approach to quantum groups is an important contribution from ring theory to Lie theory. Conversely, quasi-hereditary algebras arising from the study of highest weight categories can be regarded as a contribution from Lie theory to ring theory. In each direction, there are still several unsolved problems.

For example, q-Schur algebras (of type A) are quasi-hereditary algebras. However, similarly defined Hecke endo-algebras for types other than A share a much weaker structure. Using Kazhdan–Lusztig (KL) cell theory, B. Parshall, L. Scott and the speaker (DPS) investigated their stratification structure for those associated with finite groups

of Lie type over twenty-years ago. They also proposed a conjecture about a standard stratification via KL-cell modules. Note that these algebras played an important role in Williamson's singular Soergel bimodule theory and, more recently, in Bao–Wang's q-Schur algebras arising from quantum symmetric pairs and i-quantum groups.

I will talk about the DPS conjecture and its proof using exact categories. I will also mention applications to decomposition matrices for finite groups of Lie type in cross characteristic and to i-quantum groups.

This is joint work with B. Parshall and L. Scott.

Nan Gao (Shanghai University)

Claus Michael Ringel's main contributions to Gorenstein-projective modules

We try to recall Claus Michael Ringel's main works on the Gorenstein-projective modules. This will involve but not limited to his fundamental contributions, such as in the solution to the independence problem of totally reflexivity conditions;the technique of $\mathfrak{O}$-quivers; a fast algorithm to obtain the Gorenstein-projective modules over the Nakayama algebras; the one to one correspondence between the indecomposable non-projective perfect differential modules of a quiver and the indecomposable representations of this quiver; the description of the module category of the preprojective algebras of type $\mathbb{A}_n$ via submodule category; semi-Gorenstein-projective modules, reflexive modules, Koszul modules, as well as the Ω-growth of modules, over short local algebras; and his negative answer to the question whether an algebra has to be self-injective in case all the simple modules are reflexive.

Rasool Hafezi (Nanjing University of Information Science and Technology)

Representation theory of monomorphism categories

The study of the monomorphism category (also known as the submodule category) originates with Birkhoff and was later revitalized by Ringel and Schmidmeier, who applied Auslander- – Reiten theory to investigate its representation theory. In this talk, I will explain how the representation theory of the monomorphism category is related to that of the category of finitely presented functors over the stable category. In particular, under certain finiteness conditions, the monomorphism categories are useful in

the study of stable Auslander algebras and, more generally, stable Cohen – -Macaulay Auslander algebras. I will also discuss several generalizations of the monomorphism category. Specifically, I will classify monomorphism categories of finite representation type in terms of Dynkin diagrams. Furthermore, I will explore the structure of the monomorphism category for large modules and show how the monomorphism category of pure-injective modules relates to the study of functor categories.

Osamu Iyama (University of Tokyo)

TBA

TBA

Naihuan Jing (North Carolina State University)

q-Immanants and higher quantum Capelli identities

We construct q-immanant polynomials parameterized by Young diagrams, whose coefficients are central elements of the quantized enveloping algebra of the general linear algebra. They are shown to provide the higher quantum Capelli identities. These q-immanants generalize Drinfeld–Reshetikhin's construction of central elements in $Z(U_q(\mathfrak{g}))$ as well as q-analogues of Okounkov's quantum immanants for $U(\mathfrak{g}l(n))$. The eigenvalues of the Harish–Chandra image of the these central elements are certain factorial Schur polynomials.

This is joint work with Ming Liu and Alexander Molev.

Bernhard Keller (Université Paris Cité)

Gorenstein projective dg modules in cluster theory

Work by Geiss–Leclerc–Schröer, Pressland, Yilin Wu and others shows the importance of Gorenstein projective modules for the (additive) categorification of cluster algebras *with coefficients*. We will illustrate this point on several classical examples. We will then show how more recent examples (arising in higher Teichmuller theory) lead to the consideration of Gorenstein projective differential graded (=dg) modules over dg algebras. Dg Cohen–Macaulay modules were previously studied by Haibo Jin but the

examples from cluster theory do not quite fit into his framework since the relevant dg algebras are usually not proper.

Buyan Li (Tsinghua University)

Notes on Chevalley Groups and Root Category

Ringel used the representation theory of finite-type hereditary algebra and Hall polynomials to obtain the positive part of the simple Lie algebra. Peng and Xiao generalized Ringel' s result to root category and obtained the whole Lie algebra. Based on their constructions and following Chevalley' s method, we construct Chevalley groups from the root category. Then we prove the Bruhat' s decomposition and the simplicity of the Chevalley groups, and calculate the orders of finite Chevalley groups.

Ming Lu (Sichuan University)

Semi-derived Ringel–Hall algebras, quantum groups and i-quantum groups

Hall algebras, also known as Ringel–Hall algebras, constitute one of Ringel's most significant contributions. In 1990, he used Hall algebras to realize the half-parts of quantum groups. Ever since, Hall algebras have emerged as one of the central topics within representation theory. Inspired by Ringel's pioneering work, Lusztig, followed by Nakajima, used perverse sheaves on quiver varieties to realize quantum groups and subsequently constructed canonical bases. Bridgeland considered the Hall algebra of complexes to realize the whole quantum group. Subsequently, we introduced semi-derived Ringel–Hall algebras to rephrase Bridgeland' s result. i-quantum groups is a vast generalization of quantum groups and possess numerous appealing properties similar to those of quantum groups. We used the framework of semi-derived Ringel–Hall algebras to define i-Hall algebras on i-quiver algebras, thereby realizing i-quantum groups. As applications, we have constructed Serre presentation, braid group actions, PBW bases, Drinfeld presentations, and dual canonical bases for i-quantum groups.

In this talk, I will provide an overview of the development of Hall algebras. This presentation is based on collaborative works with Xinhong Chen, Xiaolong Pan, Liangang Peng, Shiquan Ruan, and Weiqiang Wang.

Liangang Peng (Sichuan University)

On modules determined uniquely by their dimension vectors

Let k be a field and A be a hereditary algebra over k. A well-known result said that a rigid A-module is determined uniquely by its dimension vector, namely, for two rigid A-modules M and N, if their dimension vectors are same, then $M \cong N$. The method of proving this result was to suppose the field k is an algebraically closed field and to use algebraic geometry. In this paper, we give a new proof for this result. Here, we consider any field and use a partial order given by sub-quotient modules.

Fan Qin (Beijing Normal University)

Cluster algebras, derived Hall algebras, and braid group actions

We give a gentle introduction to cluster algebras. By extending cluster algebras arising from double Bott–Samelson cells to the infinite rank setting, we recover certain infinite rank cluster algebras associated to derived Hall algebras. Their standard bases can be calculated by braid group actions.

Yu Qiu (Tsinghua University)

From green mutation to X-evolution on cluster complexes

We introduce flows and foliations on cluster complexes, generalizing green mutation on cluster exchange graphs, with application to the topology of cluster complexes. This is based on arXiv: 2501.15756 with Tang Liheng.

Shiquan Ruan (Xiamen University)

Ringel–Hall algebra approach to $i$quantum groups

In this talk, we will review several recent progress on Ringel–Hall algebra realization of $i$quantum groups. We use the iHall algebra of the category of coherent sheaves over weighted projective lines to realize the Drinfeld new presentation of universal $i$quantum groups, and use derived Hall algebras of 1-periodic derived category to realize split $i$quantum groups. Finally, we introduce a new kind of algebra associated to a hereditary abelian category, and establish the relationship with iHall algebras and derived Hall algebras. This is based on a series of joint work with Jiayi Chen, Yanan Lin, Ming Lu and Weiqiang Wang.

Markus Schmidmeier (Florida Atlantic University)

Invariant subspaces of nilpotent operators. The power of tubular algebras

Among the tame algebras introduced in the now famous Springer Lecture Notes 1099, the tubular algebras stand out for their amazingly well-organized categorical structure. Just recall that each indecomposable object is given by three coordinates, one combinatorial, one continuous and one fractional.

The power of those coordinates becomes apparent when studying in detail a particular tubular category, for example $S(6)$, the invariant subspaces of nilpotent linear operators of nilpotency index 6. Properties of objects like: "Does the subspace have a simple direct summand?" or, "does the Littlewood-Richardson tableau contain an entry 3 in its 5th row?" are specified using finitely presented functors, hence objects with the given property occur in a hammock which can be pictured in detail using the tubular coordinates.

Looking at invariant subspaces from the perspective of applications, the case of nilpotency index 6 appears artificial. So let us consider the chain of categories $S(n)$ of increasing complexity. Surprisingly, each $S(n)$ inherits the tubular coordinates from the case $n = 6$! Even in the wild cases, objects occur in tubes, hence their position within the tube is combinatorial... Yet the continuous parameters occur in spaces more interesting than projective lines... In this talk, our main interest is in the fractional coordinate which allows us to trace objects across the chain of categories.

Sibylle Schroll (Universität zu Köln)

On algebras derived equivalent to skew-gentle algebras

For classes of finite dimensional algebras, closure under derived equivalence is a rare property. By the work of Schröer and Zimmermann, the class of gentle algebras is closed under derived equivalence. On the other hand, the class of the closely related skew-gentle algebras – which are skew-group algebras of gentle algebras – is not closed under derived equivalence. In this talk, we will introduce the new class of semi-gentle algebras, given by quiver and relations. We will show that every skew-gentle algebra is semi-gentle and any semi-gentle algebra is derived equivalent to a skew-gentle algebra. This is based on joint work with Severin Barmeier and Zhengfang Wang as well as joint work in progress with Severin Barmeier, Cheol-Hyun Cho, Kyoungmo Kim, Kyungmin

Rho and Zhengfang Wang.

Gordana Todorov (Northeastern University)

TBA

TBA

Jie Xiao (Beijing Normal University)

The parity of Lusztig's restriction functor and Green's formula

Our investigation is based on three important results. (1) Ringel introduced Hall algebra for representations of a quiver over finite fields and proved the elements corresponding to simple representations satisfy the quantum Serre relations. This gives a realization of the nilpotent part of quantum group if the quiver is of finite type. (2) Green found a homological formula for the representation category of the quiver and equipped Ringel's Hall algebra with a comultiplication. The generic form of the composition subalgebra of Hall algebra generated by simple representations realizes the nilpotent part of quantum group of any type. (3) Lusztig defined induction and restriction functors for the perverse sheaves on the variety of representations of the quiver which occur in the direct images of constant sheaves on flag varieties, and he found a formula between his induction and restriction functors which gives the comultiplication as algebra homomorphism for quantum group. we prove that Lusztig's formula holds for all semisimple complexes with Weil structure. This establishes the categorification of Green's formula. This is based on the joint work of J. Fang, Y. Lan and Y. Wu.

Yu Zhou (Beijing Normal University)

τ-tilting theory in extended module categories

Let d a positive integer. We introduce and study the d-extended module category of a finite-dimensional algebra. We develop the Auslander – -Reiten theory in this setting, leading to the definition of τ-tilting pairs in the extended module category. We establish bijections between τ-tilting pairs, functorially finite positive torsion pairs, and $(d+1)$-term silting complexes. When $d = 1$, our results recover the celebrated correspondences in the τ-tilting theory for module categories as developed by Adachi, Iyama, and Reiten.