非线性分析专题研讨

时间：2026 年 1 月 14 日——1 月 19 日

地点：后主楼

组织者：李**、熊金钢

日期	时间	地点	报告人	报告题目
2026.1.14 (星期三)	09:30–10:20	后主楼 1223	罗肖雨滔 (中国科学院)	Fluid Mixing: From Norm Inflation to Inviscid Damping
	10:30–11:20	后主楼 1223	Doanh Pham (北京大学)	Monotonicity formulas for minimal submanifolds involving Mobius transformations
	14:30–16:00	后主楼 1223	Quochung Nguyen (中国科学院)	Boundary Regularity, Asymptotic Boundary Profiles, and Higher-Order Expansions in β–SQG on Bounded Domains (I)
2026.1.15 (星期四)	09:00–09:50	后主楼 1124	王聪 (对外经贸大学)	Sharp Solvability and Local Stability on the Exterior Dirichlet Problem for the Monge-Ampère Equation
	10:00–10:50	后主楼 1124	王华阳 (北京师范大学)	A Liouville Theorem for Ancient Solutions to Fast Diffusion Equations in Bounded Domains
	11:00–11:50	后主楼 1124	方俊元 (University of Tennessee)	On $W^{2,\epsilon}$ estimates for a class of singular-degenerate parabolic equations
	14:30–16:00	后主楼 1303	李**	问题研讨
2026.1.16 (星期五)	09:30–10:20	后主楼 1124	郭青 (中央民族大学)	Segregated Solutions to Critical Elliptic Systems in Higher Dimensions
	10:30–11:20	后主楼 1124	韦韡 (南京大学)	A Yamabe problem for the quotient between the Q–curvature and the scalar curvature
	14:30–16:00	后主楼 1303	李**	问题研讨
2026.1.19 (星期一)	10:00–11:30	后主楼 1124	Quochung Nguyen (中国科学院)	Boundary Regularity, Asymptotic Boundary Profiles, and Higher-Order Expansions in β–SQG on Bounded Domains (II)

2026 年 1 月 14 日(星期三)

地点：后主楼 1223

09:30—10:20

报告人：罗肖雨滔(中国科学院)

Title: Fluid Mixing: From Norm Inflation to Inviscid Damping

Abstract:

We report some recent results related to mixing in fluids. First, we demonstrate norm inflation for both the 3D Euler and Navier-Stokes equations in the supercritical regime, where mixing is the growth mechanism of such constructions. The second result concerns inviscid damping for the 2D Euler equations. We prove damping estimates for Sobolev perturbations near the Couette flow up to a long and finite time scale.

10:30—11:20

报告人：Doanh Pham(北京大学)

Title: Monotonicity formulas for minimal submanifolds involving Mobius transformations

Abstract:

Monotonicity formulas play important roles in the studies of minimal submanifolds. In this talk, I will present new monotonicity formulas for the volume of minimal submanifolds within images of concentric balls under a Möbius transformation.

14:30—16:00

报告人：Quochung Nguyen(中国科学院)

Title: Boundary Regularity, Asymptotic Boundary Profiles, and Higher-Order Expansions in β–SQG on Bounded Domains (I)

Abstract:

In these two lectures, we discuss boundary regularity issues for fractional elliptic equations, with a focus on the spectral fractional Laplacian, and introduce the generalized surface quasi-geostrophic (SQG) equation on smooth bounded domains with Dirichlet boundary conditions. After reviewing the β –SQG model and its connection to fractional elliptic operators, we explain the main analytical difficulties caused by boundary effects, which are absent in the whole-space setting.

We present an approximation scheme based on truncated fractional Laplacians and a corresponding semigroup framework, leading to local-in-time existence, uniqueness, and an interior–boundary decomposition of solutions. This approach shows how weighted norms and boundary-adapted derivatives capture the loss of regularity near the boundary $\partial\Omega$, with particular emphasis on the regime $\beta \in (0,1/2]$, where boundary singularities first arise.

In the second part, we study the asymptotic behavior of solutions near the boundary and derive sharp boundary expansions under increasing compatibility conditions on the initial data. These

results yield explicit boundary profiles involving powers of the distance to the boundary and logarithmic corrections at critical values of β, clarifying the role of geometry and fractional elliptic operators in boundary singularities.

The lectures conclude with extensions to higher regularity regimes, the case $\beta \in (1,2)$, and the generalized SQG model on the half-plane. If time permits, we will also discuss extensions to SQG with fractional dissipation and boundary regularity for certain model kinetic equations.

2026 年 1 月 15 日(星期四)

地点：后主楼 1124

09:00—09:50
报告人：王聪(对外经贸大学)

Title: Sharp Solvability and Local Stability on the Exterior Dirichlet Problem for the Monge-Ampère Equation

Abstract:

We provides two necessary and sufficient conditions for the solvability of the exterior Dirichlet problem for the Monge-Ampère equation with prescribed asymptotic behavior at infinity. Firstly, we prove that the problem is solvable if and only if the boundary value is "semi-convex", which sharpens the C^2 boundary value condition in Caffarelli-Li (2003). Secondly, under the weaker condition, we obtain a sharp threshold for solvability with respect to the asymptotic behavior at infinity. Furthermore, we establish, for the first time, the local stability of solutions with respect to perturbations in both the boundary value and the asymptotic behavior at infinity. This is a joint work with Prof. Jiguang Bao.

10:00—10:50
报告人：王华阳(北京师范大学)

Title: A Liouville Theorem for Ancient Solutions to Fast Diffusion Equations in Bounded Domains

Abstract:

In this talk, we will prove a Liouville theorem for ancient solutions to fast diffusion equations in bounded domains. To prove this theorem, we have established weighted smoothing estimates, boundary Harnack inequalities and (higher order) Schauder estimates. We shall also discuss other related open problems.

11:00—11:50
报告人：方俊元(University of Tennessee)

Title: On $W^{2,\epsilon}$ estimates for a class of singular-degenerate parabolic equations

Abstract:

In this talk, I will present recent work on weighted $W^{2,\epsilon}$ estimates for a class of parabolic

equations in non-divergence form. The leading coefficients can be singular or degenerate, or both, through a weight belonging to the Muckenhoupt class $A_{1+1/n}$. A class of weighted parabolic cylinders generically suitable for the equations will be introduced. Under some small weighted BMO assumption of the weights, a type of local quantitative lower bound estimates for solutions will also be discussed. This is joint work with Tuoc Phan (UTK).

2026 年 1 月 16 日(星期五)

地点：后主楼 1124

09:30—10:20
报告人：郭青(中央民族大学)

Title: Segregated Solutions to Critical Elliptic Systems in Higher Dimensions

Abstract:

We study the existence of multiple segregated solutions to the critical coupled Schrödinger system

$$\begin{cases} \Delta u_1 = K_1(|y|)|u_1|^{2^*-2}u_1 + \beta|u_2|^{2^*/2}|u_1|^{2^*/2-2}u_1 \ in \ \mathbb{R}^N, \\ \Delta u_2 = K_2(|y|)|u_2|^{2^*-2}u_2 + \beta|u_1|^{2^*/2}|u_2|^{2^*/2-2}u_2 \ in \ \mathbb{R}^N, \\ \qquad u_1, u_2 \geq 0, \quad u_1, u_2 \in C_0(\mathbb{R}^N), \end{cases}$$

with $N \geq 5, 2^* = 2N/(N-2)$, radial potentials $K_1, K_2 > 0$, and repulsive coupling $\beta < 0$. Under the assumption that K_1 and K_2 attain local maxima at distinct radii $r_0 \neq \rho_0$ with precise asymptotic expansions near these points, we prove the existence of infinitely many nonradial segregated solutions (u_{1k}, u_{2k}) for all sufficiently large integers k. These solutions exhibit multiple bumps concentrating on two separate circles of radii r_0 and ρ_0, respectively. Moreover, each component develops a "dead core" near the concentration points of the other. The proof overcomes the sublinear and nonsmooth nature of the coupling term (since $2^*/2 - 1 < 1$) by constructing a tailored complete metric space and combining a finite-dimensional reduction with a novel tail minimization argument. This work is joint with Zijuan Gao and Chengxiang Zhang.

10:30—11:20
报告人：韦赣(南京大学)

Title: A Yamabe problem for the quotient between the Q curvature and the scalar curvature

Abstract:

We introduce the following Yamabe problem for the quotient between the Q curvature and the scalar curvature R: Find a conformal metric g in a given conformal class $[g_0]$ with

$$Q_g/R_g = \text{const.}$$

We first prove a new Sobolev inequality between the total Q-curvature and the total scalar curvature on $\mathbb{S}^n$, for any g in the conformal class of the round metric $g_{\mathbb{S}^n}$ with positive scalar curvature, with equality if and only if g is also a metric with constant sectional curvature. With this inequality we introduce a new Yamabe constant $Y_{4,2}(M, [g_0])$ and prove the existence of the above problem provided that $Y_{4,2}(M, [g_0]) < Y_{4,2}(\mathbb{S}^n, [g_{\mathbb{S}^n}])$. This strict inequality is proved if

(M, g) is not conformally equivalent to the round sphere. Finally, we prove that on a closed n-dimensional Riemannian manifold (M, g_0) with semi-positive Q-curvature and non-negative scalar curvature, the above Yamabe problem is solvable. This is a joint work with Y. X. Ge and G. F. Wang.

2026 年 1 月 19 日(原计划为 1 月 16 日下午)

10:00—11:30 (后主楼 1124)

报告人：Quochung Nguyen(中国科学院)

Title: Boundary Regularity, Asymptotic Boundary Profiles, and Higher-Order Expansions in β–SQG on Bounded Domains (II)